The Size of the Earth

September 18, 2012

1 Important Preliminary Information

We will be taking our first field trip on the second week of this lab. We will drive to the central valley, just north of Vacaville near the town of Winters, to make our observations. It will be dark and desolate. You will need a flashlight, notebook, and pencil to do the lab. For your personal comfort you will need warm clothes just in case it gets cold, a snack and drink just in case you get hungry, perhaps toilet paper if you need to go off into the dark, and possibly insect repellent.

We will leave promptly. Class begins at 6:10 pm and soon thereafter we will be on our way. Don't be late!

2 Schedule

- *Field trip to obtain data:* Tuesday, September 18
- *Preliminary software and show-and-tell:* Tuesday, September 25
- *Report due:* Tuesday, October 2

3 Goals

Ever heard of Eratosthenes? If Nobel prizes had existed back then, he would have gotten one. Or, at least, he *should* have gotten one.

We will use his method to determine the circumference of the Earth. More specifically, we measure the difference between the elevation angles of

Polaris at two sites and the distance between the two sites. Then we combine the angle measurement (with its uncertainty) and the distance measurement (with its uncertainty) to obtain the size of the Earth (and its uncertainty).

4 Activities

We take a field trip to the Central Valley, where the weather is reliably clear and the sky is dark. We set up two observing stations on a north-south line, with two groups at each station. Each person in each group measures the altitude of Polaris at least twice. We measure the north-south distance between the stations with the odometer on the car. Think about whether there is another method that you can employ to do this more accurately! To increase the statistical accuracy, we will combine the results of all groups: the reputation of each group is at stake!

5 Data Reduction

Analyzing these data first involves a number of steps. **(1)** Properly combine the forward and reverse measurements of each theodolite (i.e., each group). **(2)** At each observing site there are two groups; make a weighted average of the two groups' results to minimize the uncertainty of the elevation angle at each site. **(3)** Combine the measurements for the two observing sites to obtain the angular difference of the zenith angle from the two sites; **(4)** combining this angular difference, with its error, together with the linear distance measurement, with its error, to obtain the Earth circumference and its error. At each stage of the process, it is important to derive not only the quantity but its uncertainty.

All this is complicated by the fact that Polaris moves, so that its elevation angle varies with time. This variation of position with time must be determined by a *least-squares fit*. This motion around the North Celestial Pole (NCP) is circular but you may be able to fit the motion with a purely linear term. This depends on the quality and quantity of data you have obtained.

There are some subtle issues involving data reduction. One is the combining of forward and reverse measurements: these will have different errors of the mean, and should you combine them with with a weighted average? Similarly, measurements of the two groups at the same site will have different errors of the mean; should you combine them with a weighted average? And measurements at the two sites will have different errors of the mean; should you combine them with a weighted average? The answer to some of these questions is “yes” and to other(s) “no”.

6 For Your Report

In addition to providing your best estimate of the Earth radius (and your error estimate), spend some time thinking about the experiment design. Why do we need to be on a North-South line? How far off a North-South line can we be and still obtain a reliable measurement? How much more accurately could you measure the Earth radius if you observed for a longer period of time? How would your experimental design and/or your results change if you could observe for 24 hours? Can you think of other ways to optimize the experiment?

7 Reference Reading

First you will need to reduce the data of your group, independently of the other groups. This is not straightforward. You can’t just combine forward and reverse to get the zenith angles, because the forward and reverse are measured at different times—and the star is moving! You need to least-squares fit each, separately, to the time, and combine the results two fits. So you need to read about. . .

6.1. Least Squares fitting. In Taylor’s book, **§8.1-8.5** discuss least-squares fitting to a straight line. This problem is made far simpler by using matrices. The matrix method is given in our tutorial entitled. . .

Least-Squares Fitting for the Budding Aficionado.
You will need to write your own software, using the method in our tuturial. *Suggestion:* when you write your least-square software, test it using the example in Taylor’s §8.5.

Next, you need to combine results of the two groups your site using the optimum statistical technique to determine the best possible result. The two groups’ results will have different uncertainties, so you need to take a weighted average. This means reading. . .

§7.1-7.3. This short chapter deals with combining measurements with different uncertainties. You face this problem when combining measurements of the same quantity by different groups.

Finally, you need to use optimum statistical techniques to determine the angular difference for the two sites and its uncertainty. Then you need to combine this difference with the distance between the two sites—and its uncertainty—to determine the circumference of the Earth. Chapter **3** tells how to combine such independent measurements and to determine the uncertainty in the result. First read §**3.3**, which tells how these combinations work and provides *provisional* equations (which you should *not* use). Then read §**3.5**, which introduces "addition in quadrature"—this is the proper way to combine uncertainties. Finally, read §**3.6** and note—and use—equations (3.16) $\rightarrow$ (3.19).