THE HI AND OH LINES, THE GALAXY, and SUPERNOVA SHOCKS

Contents

1. INTRODUCTION

Diffuse atomic gas permeates all corners of interstellar space. On the largest scales, its distribution reveals a lot about Galactic structure and dynamics. On smaller scales, its morphology is dominated by shocks from energetic stellar winds and supernovae. On still smaller scales, magnetic fields, pressure differences, and thermal instability shape the gas into thin filaments and sheets. On still smaller scales, the densities get large from self-gravitation; the gas turns molecular and stars form.

Star formation occurs in regions that are dense enough for gravity to pull together matter to form the star. In such clouds, the interstellar dust shields the inside of he cloud, eliminating the ultraviolet light that otherwise dissociates molecules. The most abundant molecule is H_2, which (owing mainly to its being a homonuclear diatomic molecule) has no spectral lines that are easily observed. However, there exist many other molecules, including simple diatomics (of which OH and CO are the most easily observed examples); triatomics (e.g. water, H_2O); polyatomics (e.g. ammonia (NH_3) and methanol (CH_4OH)), and many others. Some of these molecules radiate as astronomical masers. These include OH, H_2O, and CH_3OH. OH masers lie within the frequency range of our telescope.

For this lab, we outline several projects in HI and also one OH project that are suitable for our 4.5-m diameter Leuschner dish. The HI projects fall into two categories. One explores large-scale Galactic structure and dynamics by obtaining HI spectra along great circles, presenting the data as position-velocity images, and interpreting those maps as discussed below. The other category maps specific interstellar structures (e.g., expanding shells produced by multi-supernovae) by sampling a grid of spectra over a large area, presenting the data as grey-scale images of velocity-integrated line intensity (i.e., antenna temperature T_A) or as images in which image brightness indicates velocity-integrated T_A and color indicates velocity, and interpreting the data.

The OH project measures the spectra of a few OH masers and, also, the very broad OH in the Galactic center. We measure polarization by using crosscorrelation spectroscopy to measure all four Stokes parameters simultaneously.

We'd like each group to pick a single project and carry it through to completion by the time the lab reports are due, which is the last day of instruction (May 5). All of the HI projects involve making maps and images. One of the features of this lab is to learn about image processing and using colors in images.

For the great-circle projects (§4 and §6), we gain large angular scale information in the sky by observing on great circles. Getting nice results requires data over the largest portions of the great circles you can get. For a full 360-degree circle, that's 180 pointings at 2° spacings. Mostly, though, we can't cover a full great circle because some of the circle lies too far south, so we do as much as we can.

2. SCHEDULE

The end of the semester is approaching like a freight train. And our new telescope at Leuschner isn't even functional yet! Imagine yourself tied onto the tracks by a madman: you've gotta finish before it hits. And the madman won't let you even begin serious measurements before the dish is ready! Here's a schedule. If we don't keep up with this by making the telescope work during the first week, and if you don't keep up with this, we're in trouble!

1. Week 1, 08apr-14apr: Obtain and reduce test spectra from our new spectrometer. Develop software so that a properly calibrated spectrum (intensity calibration and Doppler calibrated/corrected) can be created from two frequency-switched spectra in a single procedure call. When the telescope is finally pointable, take the data and use this software to make a plot of the completely calibrated HI line spectrum at the position in Galactic coordinates $(\ell, b) = (120°, 0°)$. We will compare the spectra of all groups at the class on 14 April, and if your group doesn't have a spectrum to show this will be considered very bad form.

   Pick a group project. Figure out the RA and Dec boundaries of your object, when the object is up, and how you will get the data for your project. Coordinate with other groups to figure out how you can share telescope time. With luck, the telescope will be working by the end of the week. Be prepared for this circumstance, and if it comes to pass, get started on your project!

2. Week 2, 15apr-21apr: Get as much data as you can. Reduce the data. If you have enough data, begin the map-making process.

3. Week 3, 22apr-28apr: Get the rest of your data. Figure out how you want to present the data as images. Make some first-try images.

4. Week 4, 16apr-03may: Finalize data. Make final images. Write report. Report is due the last day of class, 05may.

3. ON MAPPING AREAS: HOW TO SPATIALLY SAMPLE

All of the HI projects examine the angular dependence of the 21-cm line profile, so you need closely-spaced observations to make a map or an image. How closely should the observations be spaced? The usual criterion is two samples per HPBW (Half-Power BeamWidth), because observing at more closely spaced angles just gives redundant information and takes extra time, while more widely spaced angles loses small-scale structural information. (This statement can be justified more exactly by Fourier transforming the structure of the sky in terms of angular frequency and then imposing the Nyquist criterion...). The HPBW is $\sim 3°$; using a somewhat larger spacing of $2°$ makes things go faster and it's being a round number is nice.

For the mapping projects (not the great-circle ones), we need to cover areas of 1000 to 2000 square degrees. Consider the Orion-Eridanus Superbubble (§7 below), which wants to map the the region covering roughly ($\ell = 160° \to 220°, b = -70° \to -10°$). You might think you'd set up a regular grid in (ℓ, b) with ($\Delta\ell, \Delta b$) both equal to $2°$. That's find for Δb, but you'd be sampling too closely in ℓ. That's because the true (great-circle) angular distance between two points is $\Delta\ell \cdot \cos(b)$; there's a foreshortening for $b \neq 0°$ (e.g.: when you're at the Earth's North Pole, you can go completely around a small circle of constant b with just a few steps.). So you can save lots of observing time by observing at $\Delta\ell = \frac{2°}{\cos(b)}$ instead of $\Delta\ell = 2°$.

The true angular area is not the total extent in ℓ times the total extent in b because of the foreshortening discussed in the above paragraph. So the actual angular area of the Orion-Eridanus area area isn't the range in ℓ times the range in b, i.e. $60° \times 60° = 3600$ square degrees (which would require 900 pointings), but rather only about 2800 square degrees. At $2°$ sampling, that's about 700 pointings.

These projects require what seems like a lot of pointings, but—hey!—the telescope is automated. (With luck, that is...) With the magic of a computer-controlled telescope you can do these observations in just one or two days. But it will require planning and coordination with other groups—all of you are in the same boat! (Or should we say, "in the same dish"?)

4. THE GALACTIC PLANE: THE GREAT CIRCLE AT LATITUDE ZERO ($b = 0°$)

Point the telescope at a series of Galactic longitudes (ℓ) along the Galactic plane (Galactic latitude $b = 0°$), over the full longitude range that is observable for our telescope. Getting nice results requires data over the largest section of the Galactic plane you can get, which is roughly (or somewhat less than) $\sim -10° \lesssim \ell \lesssim 250°$. This observation has two science goals:

1. *The Galactic rotation curve.* Determine the rotation curve of the Galaxy for the portion of the Galaxy inside the Sun and use the result to estimate the *gravitational* mass M_{grav} of the Galaxy that lies inside the Solar circle; for this, you just assume the usual $v^2 = \frac{GM_{grav}}{R}$. In fact, you can even get the radial dependence $M_{grav}(R)$! Also, use your observations to estimate the *gaseous* mass $M_{gas}(R)$ of the Galaxy. What fraction of the total Galactic mass comes from interstellar gas? For this you'll need to know the radius of the Solar circle: it's 8.5 kpc. See *Commentary* below for a discussion.

2. *The Galactic center.* Our Galactic center, like others, has a big black hole, with infalling matter and jets and lots of anamolous velocities. You can see a lot of this activity in your map. Using the spatial extents and the velocities, you can estimate a black hole mass. To get an accurate estimate, you need to look at motions of matter lying really close to the black hole, which you can't do because you don't have the angular resolution. But you can at least

make an estimate!

3. *Spiral Structure of the Galaxy.* All your life, you've been told that we live in a spiral galaxy. Are they lying? In principle, to detect spiral structure you make a position-velocity plot along the Galactic plane [that's the image $T_A(\ell, V_{LSR})$ for $b = 0°$]. Bright regions are regions of excess density—or regions in which lots of HI is packed into a narrow velocity range (see §12.1 below).

   Do you see spirals in your position-velocity plot? How do you tell? There's only one easy way. Make a model of a spiral arm and see how it projects onto position-velocity space. To do this, you need to know the rotation curve. You've measured it for the inner Galaxy; for the outer Galaxy (outside the Solar circle) assume that the rotation curve is constant beyond the solar circle [$V(R)$=220 km/s for $R > 8.5$ kpc]. In polar coordinates, the equation for a spiral arm is

$$R_{arm} = R_0 e^{\kappa(\phi - \phi_0)} \tag{1}$$

   where R_0, κ, and ϕ_0 are free parameters; κ is the tangent of the *pitch angle.* You project this into position-velocity space using equation 8 and your knowledge of the Galactic geometry.

   Comment: Finding spiral arms inside the Solar circle is very difficult, and has thwarted efforts by astronomers over the past several decades. Finding them in the outer Galaxy is not so bad.

5. IS THE GALACTIC PLANE FLAT?

External galaxies are often distorted (like the brim of a Fedora). These distortions are probably produced primarily by nearby neighbors. How about our own Galaxy? To determine this we want to look at the vertical structure of the Galactic plane and see where the peaks occur; a flat Galaxy would everywhere have the peaks at $b = 0°$.

The most comprehensive way to observe and display these distortions is with a map that covers the biggest possible swath in Galactic longitude ℓ for the range of Galactic latitude (b) from $-20°$ to $+20°$. This is a huge area—almost 10^4 square degrees! At $2°$ sampling, you'd need ~ 2500 profiles. To make the project more manageable, you can increase the increment in ℓ, say to 4 degrees, while keeping the same, fully-sampled increment in b. Resolution is l is less important because—like a fedora—the warp doesn't change rapidly with l.

This project affords a great opportunity to make a spectacular color image in (ℓ, b) coordinates: brightness showing the amount of gas, color the Galactic rotation. Officially, the science goals here are to estimate the thicknesses of the Galactic disk and, also, to obtain an approximate Fourier representation of the warp—which is not so straightforward given the absence of data in the "Southern" Galaxy.

6. THE GALACTIC POLES: GREAT CIRCLES AT CONSTANT LONGITUDE ℓ

How is the interstellar gas distributed within the Galaxy—for example, is it "disklike" or more spherically distributed? Are there any systematic motions other than just "Galactic rotation"? (You'd be surprised!). To answer these questions, determine the column density and mean velocity of interstellar gas as a function of Galactic latitude b.

To answer this completely you'd need to do a sky survey! That's a big job. So let's do something easier. We have one project (§4) that looks around a great circle, the Galactic equator (constant b). Other projects could check out one or more orthogonal great circles (at constant longitudes ℓ) going from one Galactic pole to the other and back again (or, if not the whole 360 degree circle, as much as possible). Particularly appropriate sets of constant-longitude great circles are the pairs $\ell = (220°, 40°)$ (partly because you can get the whole great circle) and $\ell = (130°, 310°)$ (because at positive latitudes there's weak high-velocity gas).

The science goals here are to characterize the thickness of the gas layer and to determine the vertical kinematics of the gas, and if possible an estimate of the energy involved in the vertical motions.

7. MAPPING THE ORION-ERIDANUS SUPERBUBBLE

The "Orion-Eridanus Superbubble" was, and continues to be, produced by energetic stellar winds and supernovae that were located in immediate vicinity of the Orion nebula. For this, you'd map the region covering roughly $(\ell = 160° \rightarrow 220°, b = -70° \rightarrow -10°)$. Interesting features should produce antenna temperatures ~ 20 K. This is almost 2800 square degrees and, with 2 degree spacing, requires almost 700 profiles. See §3!

The goal here is to map the HI in the 3-d space of (l, b, v) (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the swept-up gas piled up at the edges.

8. MAPPING THE NORTH POLAR SPUR and its associated EXPANDING HI SHELL

This is a huge shell produced (and continuing) by several energetic stellar winds and supernovae that were located in the Sco/Oph association of stars. The shell contains not only HI but also relativistic electrons, so the shell is visible in both the 21-cm line and synchrotron emission. We can't easily see the latter, but we can see the HI—and the pattern of its velocity follows that of an expanding shell.

For this, you'd map the region covering roughly $(\ell = 210° \rightarrow 20°, b = 0° \rightarrow 90°)$. (The

longitude range is 170°: 210° → 360° plus 0° → 20°). This is roughly 1/4 of the whole sky, so this is a huge area—almost 10000 square degrees! (This shell has an angular diameter of 120°). But (unfortunately) you can't measure the equivalent 2000 HI profiles because most of this area is too far south. At least, this makes the project do-able! In doing your observations, you need to cover as far south as you possibly can. See §3!

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

9. MAPPING A BIG HIGH-VELOCITY CLOUD

When we look up, away from the Galactic plane, we see infalling gas—some a very high velocities. It's called "High-Velocity Gas". The line is weak (about 1 to 1.5 K), so you need high sensitivity—you need to use much longer integration times than for the above projects, at least a few minutes per point. This project needs to map the region bounded roughly by ($\ell = 60° \rightarrow 180°$), $b = 20° \rightarrow 60°$). This is about 3700 square degrees and needs about 900 profiles at 2° spacing. See §3!

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

10. MAPPING THE MAGELLANIC STREAM

The Magellanic Stream is an intergalactic tidal stream produced by gravitational interaction between the Magellanic Clouds and the Milky Way. The signal is weak ($\sim$ a few tenths K), so you need to use much longer integration times than for the above projects—a minimum of 10 minutes per point. You need to map the pie-shaped area bounded roughly by ($\ell = 60° \rightarrow 110°$), $b = -90° \rightarrow -30°$). This is about 1250 square degrees and needs about 310 profiles at 2° spacing. See §3! Also, the LSR velocity of this gas ranges from roughly -400 to -100 km/s along its length, so you need frequency switch by a large enough interval!

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

11.   THE OH MOLECULE

This project is more oriented towards computing and signal processing than "astronomy", and uses cross correlation to derive the complete set of polarization Stokes parameters. It doesn't use images, except possibly for interference detection and excision, and as such would not appeal to everyone.

The OH molecule is an important one for star formation and molecular clouds because it is one of the first to form when a diffuse atomic cloud starts to become denser; in other words, OH marks the transition between atomic and molecular gas. It persists at higher densities. The ground rotational/vibrational state exhibits "Λ doubling", which produces two spectral lines; each of these is hyperfine-split (just like the 21-cm line), and the net result is four lines centered at 1612.2310, 1665.4018, 1667.3590, 1720.5300 MHz. At sufficiently high densities, OH can be "pumped" by collisions, IR, or UV radiation to make these lines shine as masers. The strongest maser line tends to be the 1665 line. 1667 is the strongest nonmaser line.

The maser lines are strong, but our telescope is small, and the strongest source, W49, will produce < 1 K antenna temperature. We have interference, which makes the line difficult—maybe impossible—to detect. We have never been able to detect it, but with our new dish and spectrometer it should be do-able.

The goal here is to detect the 1665 MHz maser line and to measure its velocities and complete state of polarization in a number of HII regions, where it appears in maser emission, and towards the Galactic center, where it appears in very broad absorption. The HII regions are W3, W33, W43, and W49. We will use W49 as a polarization calibrator, because one of its velocity components is 100% circularly polarized, and having that calibration measure the polarization state of the other velocity components in W49 and, also, in the other HII regions. The coordinates of these HII regions are in the *Simbad Astronomical Database*[1]. While you're at it, also measure the OH line towards the Galactic center. It should show up as broad absorption lines against the continuum of the Galactic center.

12.   BASICS OF THE 21-CM LINE.

12.1.   Column Density and Mass

The 21-cm line, with frequency 1420.405751786 MHz, comes from atomic hydrogen (HI). In the terrestrial environment, H atoms quickly become H_2 molecules; in interstellar space, where densities are far lower than in the best vacuum systems on Earth and there are "lots" of UV photons that dissociate H_2, H remains atomic unless it resides inside dark clouds where it is shielded from

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[1]To reach *Simbad*, Google it, and once you reach the site click on 'by identifier'.

starlight.

The intensity of the 21-cm line is directly proportional to the column density of H atoms as long as the opacity of the line is small; this is a reasonably good approximation (but not perfect, particularly in the Galactic plane where the line is strong). With this approximation,

$$N_{HI} = 1.8 \times 10^{18} \int T_B(v) dv \ \text{cm}^{-2}. \tag{2}$$

Here N_{HI} is the column density of H atoms—the number of atoms in a 1 cm^2 column along the line of sight; $T_B(v)$ is the brightness temperature of the 21-cm line, which is a function of velocity v; and v is the velocity in km s^{-1}. The velocity v is produced by the Doppler effect, so a frequency shift $\Delta\nu$ from line center corresponds to velocity $v = -c\frac{\Delta\nu}{\nu}$ or, for the 21-cm line, $v = -\frac{\Delta\nu}{4.73\text{kHz}}$. Note the minus sign! It means that positive velocities are receding—a very convenient convention for astronomy, because of the expansion of the Universe.

Note an important corollary to equation 2: If we consider a small velocity interval Δv, then the number of H atoms in the column in that velocity range is just

$$N_{HI}(v \to v + \Delta v) = 1.8 \times 10^{18} T_B(v) \Delta v \ \text{cm}^{-2} \ . \tag{3}$$

This is very important, because it gives us the opportunity to make maps of the interstellar gas at different velocities and to determine how it moves.

To get the column density we need to get the brightness temperature T_B, which is not the same as our antenna temperature T_A. The relationship between these depends on the relative size of the source and the beam. This is equation (9) in the "Fount of All Knowledge!" handout, namely

$$T_A \approx T_B \frac{\Omega_s}{\Omega_s + \Omega_b} \ , \tag{4}$$

where Ω_s and Ω_b are the solid angles of the source and beam, respectively. From this, we see that if the source fills the beam ($\Omega_s \gtrsim \Omega_b$) then $T_A \approx \langle T_B \rangle$ (the brackets denote the average over solid angle); while if it is much smaller ($\Omega_s \ll \Omega_b$) then $T_A \approx \frac{\langle T_B \Omega_s \rangle}{\Omega_b}$, i.e. T_A is smaller by the ratio of the solid angles.

To get the mass of HI from the column density, you need to know the area of the region in *linear* measure, which is just $\Omega \, d^2$, where d is the distance. Each HI atom has mass m_H, so the total mass of HI in a region of size Ω is just

$$M_{HI} = m_H \, d^2 \, \langle N_{HI}\Omega \rangle \ . \tag{5a}$$

Here $\langle N_{HI}\Omega \rangle$ is shorthand for the general relation, which uses an integral:

$$\langle N_{HI}\Omega \rangle = \int_{\text{region}} N_{HI} d\Omega \qquad (5b)$$

Substituting for N_{HI} from equation 3, we have

$$M_{HI}(v) = 1.8 \times 10^{18} \Delta v \ d^2 \ m_H \ \langle T_B\Omega \rangle \ \text{gm} . \qquad (6)$$

Units are cgs. Sometimes there is a well-defined source with boundaries; in this case, to get the mass of the whole source, we use the solid angle occupied by the source, Ω_s.

Sometimes (in fact, often) we want to know the mass seen *by the telescope* for some particular observation. For an extended region with $\Omega \gg \Omega_b$, $T_A = T_B$ and the telescope sees the solid angle Ω_b, so the product $\langle T_B\Omega \rangle = T_A\Omega_b$ because angular area seen by the telescope is limited by its beam size. In contrast, if the HI emission is limited to a small region—a small source with $\Omega_s \ll \Omega_b$—then $T_A \approx \frac{\langle T_B\Omega_b \rangle}{\Omega_s}$ so the product $\langle T_B\Omega_s \rangle = T_A\Omega_b$; the telescope sees the whole source.

For both extremes, the product $T_B\Omega = T_A\Omega_b$. It's not only these two extremes, but for *all* sources the product $\langle T_B\Omega \rangle = T_A\Omega_b$. Thus, it is always true that the mass seen by the telescope is

$$M_{HI}(v) = 1.8 \times 10^{18} \Delta v \ d^2 \ m_H \ T_A\Omega_b \ \text{gm} . \qquad (7)$$

The distance d is usually a function of velocity v, especially in the Galactic plane.

$T_A(v)$ is directly measured and Ω_b is the telescope beam area, which is known, so it's easy to calculate $M_{HI}(v)$—but only if you know the distance! Often you don't know the distance, but you might have a hunch for a reasonable value for the distance. Suppose this is 100 parsecs. In this case, people usually evaluate the mass for this distance and, when giving the mass, say "The mass is $xxx \times d_{100}$, where d_{100} is the distance in units of 100 pc".

12.2. Converting Galactic Rotation to Doppler Velocity

Measurement of the Doppler shift caused by differential Galactic rotation for $0° < \ell < 90°$ allows a direct determination of the Galactic rotation curve inside the Solar circle, using the "tangent point" method. See Burton's article "Structure of our Galaxy from Observations of HI" in *Galactic and Extragalactic Radio Astronomy, second edition* (editors: G.L. Verschuur and K.I. Kellermann). In essence, you use the following equation.

We won't go through the derivation here; we cover it in class, and it is also done in Burton's article, page 303-304. Let V_{Dopp} be the Doppler velocity (this is also the observed LSR velocity), $V(R)$ be the Galactic rotation velocity at Galactocentric distance R, and let the subscript $\odot$ denote

values at the Solar circle. Then we have

$$V_{Dopp} = \left[\frac{V(R)}{R} - \frac{V(R_\odot)}{R_\odot} \right] R_\odot \sin(\ell) \tag{8}$$

The values at the solar circle are $V(R_\odot) \approx 220$ km/s, $R_\odot \approx 8.5$ kpc.

13.   ON MEASURING POLARIZATION

Polarization is completely specified by the Stokes parameters. Stokes parameters are linear combinations of power measured in *orthogonal polarizations*. There are four:

$$I = E_X^2 + E_Y^2 = E_{0°}^2 + E_{90°}^2 \tag{9a}$$

$$Q = E_X^2 - E_Y^2 = E_{0°}^2 - E_{90°}^2 \tag{9b}$$

$$U = E_{45°}^2 - E_{-45°}^2 \tag{9c}$$

$$V = E_{LCP}^2 - E_{RCP}^2 \tag{9d}$$

We like to write the *Stokes vector*

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}.$$

The first, Stokes I, is total intensity. It is the sum of *any two orthogonal* polarizations. The second two, Stokes Q and U, completely specify linear polarization. The last, Stokes V, completely specifies circular polarization. One often specifies fractional polarized power instead of polarized power itself, by dividing all Stokes parameters by I; in this case, of course, $I = 1$.

As shown in Figure 1, left panel, linear polarization is conventionally specified by two different parameters, the polarized intensity P_{QU} and the position angle χ. These are directly related to the Q ande U Stokes parameters:

$$P_{QU} = (Q^2 + U^2)^{1/2} \tag{10a}$$

$$\chi = 0.5 \tan^{-1} \frac{U}{Q} \tag{10b}$$

with the inverse relationships

$$Q = P_{QU} \cos(2\chi) \tag{11a}$$

$$U = P_{QU} \sin(2\chi) \tag{11b}$$
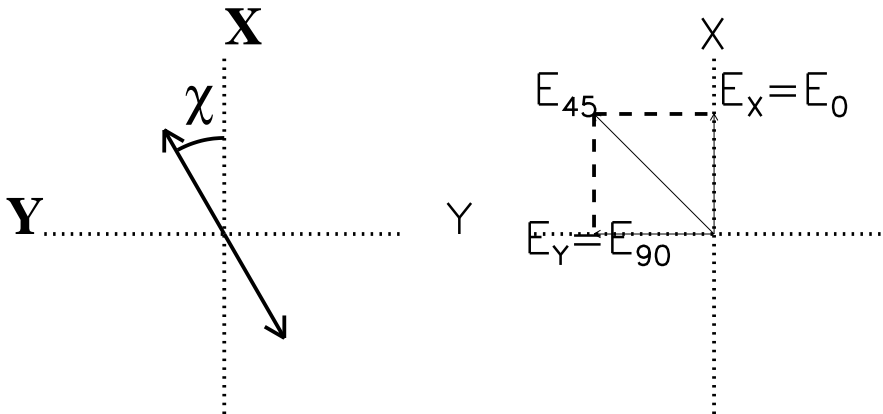

Fig. 1.— Left: The position angle of polarization. Right: Adding E_X (same as E_0) and E_Y (E_{90}) gives E_{45}.

Real radioastronomers measure all four Stokes parameters simultaneously! After all, extracting two orthogonal polarizations provides *all* the information; you can synthesize *all* other E fields from the two measured ones! Example (Figure 1, right panel): Sample (E_X, E_Y) and synthesize E_{45} from (E_X, E_Y). To generate E_{45}, add (E_X, E_Y) with no phase difference. To generate E_{LCP}, add (E_X, E_Y) with a 90° phase difference.

Carrying through the algebra for the two linears: It's clear that

$$E_{45°} = \frac{E_{0°} + E_{90°}}{\sqrt{2}} \tag{12a}$$

$$E_{-45°} = \frac{E_{0°} - E_{90°}}{\sqrt{2}} \tag{12b}$$

Write the two linear Stokes parameters:

$$Q = E_X^2 - E_Y^2 = E_{0°}^2 - E_{90°}^2$$

$$U = E_{45°}^2 - E_{-45°}^2 = 2 E_X E_Y$$

Stokes U is given by the crosscorrelation $E_X E_Y$. To get V, throw a 90° phase factor into the correlation. Our new spectromer provides this cross-product, so this means we can be *real* radio astronomers! We have a handout explaining all the gory details for any group that pursues this project.