ASTRONOMY WITH THE 21-CM LINE; SOME MICROWAVE ELECTRONICS
February 7, 2017

Contents

1. GOALS

- Understand time: UT/UTC, PST, LST, Julian day.

- Measure a 21-cm line power spectrum from atomic Hydrogen in the Milky Way.

- Combine data by averaging, medianing.

- Remove the instrumental contribution to the spectrum.

- Use rotation matrices to convert among various spherical coordinate systems.

- Produce spectra with velocities expressed in various coordinate systems.

- Representing spectra by Gaussian components.

- Learn the basics of transmission lines and waveguides.

- Learn about reflections and minimizing them with impedance matching.

- Derive propagation velocities by measuring the wavelength of standing waves.

- Empirically explore waveguides and their cutoff properties.

- Use linear and nonlinear least-squares fitting to squeeze the best results from your data.

2. SCHEDULE

1. *The First Week. For class JD=2457799.583:* Finish §6 and §7, and understand §4. Be prepared for 'show and tell'.

2. *The Second Week. For class JD=24577806.583:* Finish §8, §9, and §10.

3. *The Third Week. For class JD=2457813.583:* Write up your formal report. And finish what you couldn't get done during the first two weeks!

3. YOUR LAB REPORT

The lab reports, as always, are written by *individuals*.

The microwave and other electronics is such an important part of this lab that you should include a block diagram of the telescope electronics in your lab report. You can prepare this either by either hand or computer. If by hand, you'll have to make a PostScript file so you can insert into your report's tex file. If by computer, you can use your favorite software for making line drawings or, if you want to become a Linux guru, use `xfig`, a menu-driven program with ancient written documentation in `/home/global/ay121/handouts/xfigbookps` (or, of course, google `xfig` on the web). Tex requires PostScript images (`*.ps`) for its figures. and you can convert among file types using the Linux `convert` facility (e.g. `convert myfile.jpg myfile.ps`) or, better for `*.pdf` to `*.ps` or vice-versa, use `pdf2ps myfile.pdf` or `ps2pdf myfile.ps`.

The real work in this lab is the analysis, so it provides the ideal opportunity for you to flex your programming muscles. Accordingly, *each individual* needs to write her/his own software. And this software needs to be documented, preferably using the IDL standard format (see, for example, the documentation for our procedure that gives LST (by typing `doc, 'ilst'` at the IDL prompt). *Include this documentation* (but not the rest of the software) *as part of your lab report.* Also include a brief description of how to run your

software; tell me the subdirectory where your software can be found; and make sure I can look at those subdirectories by setting the permissions so that they are readable by 'world'.

4. WHAT TIME IS IT?

4.1. Whaddya Mean by Time?

Coordinated Universal Time (UTC) is the Civil Time[1] in Greenwich, England, which is 8 hours ahead of (larger than) PST. Remember that in the middle of this course, we switch from PST to PDT; PST=UTC - 8 hr, while PDT=UTC - 7 hr). Think of Civil time as Solar time, because the 24-hour period is the same as that of the Sun.

Think of Local Sidereal Time as "star time"; it tells when a particular star (not the Sun) is visible. The 24-hour LST period is a bit shorter than the 24-hour Solar time period—$\frac{1}{356}$ less, to be almost exact. This is a little less than 4 minutes per day.

The Julian Day is a sequential numbering of Solar days. As such, it uniquely specifies the date without the need for any complications such as months (with their nonsensical definitions[2]), leap years, etc. It begins at noon (UT=12hr) in Greenwich, so it is 12 hours out of phase with UT. But the international date line is 12 hours away from UT, so the JD is in sync with humankind's definition of 'when the day begins'. JD=0 begins at Greenwich noon on 1 Jan –4713. The Julian day is a double-precision float: it contains the time as the fractional part of the day. It's not just an integer day number. The current JD is about 2457793.

The Modified Julian Day is a shorter version of Julian Day. MJD is 12 hours out of sync with JD, so it is in sync with local time at Greenwich. MJD=0 occurs at 0 hr UT on 17 Nov 1858. The current MJD is about 57792.

4.2. Software—Some Useful IDL Procedures for Time Conversion

These routines are based on GSFC `ct2lst.pro`. LST tells when a star is overhead, so its rate of passage depends on the Earth's spin. The Earth is constantly slowing owing to tidal friction produced by the Moon. The conversion between Civil Time and LST has to

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[1] You use Civil Time when you set your alarm clock for getting up in the morning.

[2] "Thirty days hath September. . . "

be adjusted periodically by inserting 'leap seconds', so unless you keep up with this, the conversion will be off by a few seconds.

You need to know your longitude to calculate LST. NCH has
nlat=37.8732, elong=-122.2573 = (360.-122.2573) (same as wlong=+122.2573).
For all procedures mentioned here, these are the defaults.

```
local_now = SYSTIME() gives current local time as a string
```

```
ut_now = SYSTIME(/ut) gives current UTC as a string
```

```
ut_sec_since = SYSTIME(/sec) gives seconds elapsed since 1 January 1970
UTC
```

```
julian_now = SYSTIME(/julian, /utc) gives current julian day (which
        contains the current time, too--it's not just an integer
        number. the /utc is required for it to give julian date relative
        to Greenwich, which is the only one that means anything.
```

```
LST -- prints LST right now on the screen
```

```
lst_now = LSTNOW() -- returns the current LST
```

```
lst_julian = ILST( julian=juldate) -- returns the LST for the specified
        Julian day
```

```
CALDAT, Julian, Month [, Day [, Year [, Hour [, Minute [, Second]]]]]
        Given the Julian day (which is defined relative to Greenwich),
        this returns the civil time at Greenwich.
```

```
julian = JULDAY(Month, Day, Year, Hour, Minute, Second)
        Given the civil time at Greenwich, this returns the Julian day.
```

```
modified_julian_day = MJD2JD( julian_day, /reverse)
```

```
julian_day = MJD2JD( modified_julian_day)
```

5. HANDOUTS

1. A second-level IDL tutorial. `idldatatypes.pdf` "QUICK IDL TUTORIAL NUMBER TWO: IDL DATATYPES AND ORGANIZATIONAL STRUCTURES" For this lab you use complex numbers, matrices and matrix math, and structures. This handout describes the complete set and, in addition, how numbers are stored and represented, and how accurate they are. *You need to know all of this stuff!*

2. The complete story of producing calibrated spectral line shapes and intensities:"CALIBRATING THE INTENSITY AND SHAPE OF SPECTRAL LINES". This handout is *optional* because the steps discussed in this lab writeup (what you're reading now) are sufficient, but if they are unclear or you want to know more, this writeup has it all.

3. What physical quantity are we measuring? Sure, it's power received by the telescope. But what does that mean about the source being observed? It's all about specific intensity, usually denoted I: "SPECIFIC INTENSITY: THE FOUNT OF ALL KNOWLEDGE! or I, T_B, T_{SYS}, T_{RCVR}, S, and $\tau \Delta \nu$. You need to understand these topics to interpret your measurements.

4. Astronomical coordinate transformations with Rotation Matrices: `ay120coord.pdf` "AY120A,B CHEAT-SHEET FOR SPHERICAL COORDINATE TRANSFORMATION". converting among Az/El, Ha/Dec, Ra/Dec, L/B. *You need to know all of this stuff!*

5. You can do this lab without doing any textbook-type reading... but then you wouldn't gain the insight from the highly readable and well-written text by Ramo, Whinnery, and van Duzer (RWvD), *Fields and Waves in Communication Electronics*. This is a great book because it contains commentary on applications to modern devices (e.g., the discussions of transmission lines carrying pulses from one piece of computer hardware to another are very nice). Reading this book is *optional*, but in any case see notes below in §11.

6. IN THE LAB: YOUR FIRST 21-cm LINE MEASUREMENT (First Week; by GROUPS)

This week we will use the techniques we explored in Lab 1 to measure the 21-cm line (colloquially known among astronomers as the *HI*—that's *H - "Roman numeral I", pronounced "H-one"*) line shape, velocity, and intensity using the big horn on Campbell's roof.

Most of the measured power comes from our own electronics, not the HI line. It's often called "noise", and we need to get rid of this instrumental contribution. We also need obtain the correct line shape and intensity.

We'll measure the 21-cm line twice. For the first, the goal is to master the technical aspects and familiarize ourselves with the system and procedures, so instead of worrying about where to point the horn we'll just take whatever position happens to be overhead. For the second (§8), we'll manually point the horn to a designated position and make a calibrated profile to compare with a well-established standard profile measurement.

6.1. The Receiving System

We will use a double-heterodyne system for the 21-cm line at 1420.4 MHz. 'Double heterodyne' means we have two mixing stages. It works like this:

1. For the first mixer, we use the ordinary DSB technique. Suppose we use an l.o. (the 'first local oscillator') at 1230.0 MHz so that the difference frequency is 190.4 MHz and the sum frequency is 2650.4 MHz. We use a 20 MHz-wide bandpass filter centered at 190 MHz to obliterate the sum frequency, so we are left with a replica of the 21-cm line that is centered at 190.4 MHz instead of 1420.4 MHz.

2. For the second mixer, we use the SSB technique. The second l.o. has frequency 190.0 MHz, so the output frequencies are positive and negative, centered about zero; this is called 'baseband'. In the absence of Doppler shift, we are left with the line centered at 0.4 MHz. However, the line is shifted and broadened by Galactic rotation and the Earth's orbital velocity, so it covers a range of frequency of, for example, 1420.2 ± 0.5 MHz. We use a low-pass filter with a cutoff frequency of, say, 2 MHz to eliminate aliasing, sample the complex signal, and Fourier transform to calculate the power spectrum. We take many power spectra and average them to reduce the noise.

6.2. Your Measurement

Before taking actual data, it's important to do some basic system checks: (1) make sure the signal levels are OK and (2) experimentally determine which way frequency increases on your power spectra.

1. Set the system up as you will be using it to observe. Point the horn to zenith to reduce interference and thermal noise. Take some data. How fast must you sample?

2. Look at the range of sample values by plotting a bunch of them. Best is to make a histogram, which you can do using IDL's `histogram` function. We have a convenient 'wrapper' called `histo_wrap`, which you may find easier to use[3]. The sampled numbers should cover plenty of bits; quantization should be only barely, or not at all, visibly evident in the histogram. Adjust `voltrange` in the call to `getpico` accordingly.

   The histogram shape should look like a well-known function. Which function? Does it?

3. Insert a test signal so that it appears in the upper sideband and take some data; then change the test signal frequency so it's in the lower sideband. You'll use this to make sure that the SSB mixer is doing its job properly, and to determine whether the frequency axis is flipped or not.

4. The horn is a gigantic rain bucket. You won't see any astronomical signal unless you dump it!

Having determined that the system basics work, it's time to do astronomy! For this first measurement the main goal is to master the technical aspects, so use whatever position happens to be overhead and point the horn straight up. The line will be strongest—strong enough to see visually—for the approximate range LST = 19 - 6 hr.

It is most convenient to use temperature units—Kelvins—for the power that we measure. Accordingly, the power that we measure is called the 'System Temperature' T_{sys}. It's a function of frequency and has two kinds of behavior: the 'continuum', which is devoid of spectral features and changes very slowly with frequency; and the 'line', which in this case is the 21-cm line and it changes rapidly with frequency—hence our desire to obtain the line shape.

The System Temperature has two contributions: (1) the dominant contribution from our electronics, which we call the 'Receiver Temperature' T_R; and (2) the contribution from sky, being picked up by the antenna, the 'Antenna Temperature' T_A. Thus $T_{sys} = T_R + T_A$, and usually $T_R \gg T_A$.

Our horn is equipped with an old, noisy first amplifier so $T_R \sim 300$ K; in contrast, our Leuschner telescope is much better, with $T_R \sim 50$ K. The antenna temperature comes

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[3]`histogram` is a native IDL function, so to get its documentation type `?histogram`. In contrast, `histo_wrap` is our own, in-house creation; for documentation of such non-native IDL functions, type `doc,'histo_wrap'`. (There's another wrapper for plotting called `hist`, but we find it too complicated.)

from the Cosmic Background, with brightness temperature ($T_{B,CBR} = 2.8$ K); from interstellar/intergalactic space with brightness temperature $T_{B,astron}$ (usually no more than a few K in the continuum and up to about 100 K in the HI line); and the Earth's atmosphere with brightness temperature $T_{B,atm}$, perhaps a few K at the HI line frequency. So off the HI line we have $T_A \sim 10$ K and on the HI line, in the Galactic plane where it is strongest, we have $T_A \sim 100$ K.

We'll take two sets of data: (1) a long integration to measure the line *shape.* and (2) a short integration so we can calibrate the line's *intensity.*

1. The measured power spectrum *shape* is dominated by the frequency-limiting filters acting on the System Temperature. To see the line, which is weak, we need to correct for these filter shapes, which we do by obtaining a spectrum containing no line. Accordingly, to get the shape we take two spectra: one with the line present (the 'online' spectrum s_{online}) and one with the line not present (the 'offline' spectrum $s_{offline}$).

   We could obtain the online spectrum by centering the line and making a measurement; then changing the first l.o. frequency so that the line shifts either completely outside the band (this is called 'frequency switching'), or partway over but is still in the band (called 'in-band frequency switching'). The latter is better because you are always looking at the line, thus getting better sig/noise.

   To accomplish this, take a spectrum with the line roughly in the upper half of the baseband spectrum, and another with it centered roughly in the lower half. Use the first as the 'online' and the second as the 'offline' spectrum for the upper half. Similarly, for the lower half, use the second as the 'online' and the first for the 'offline'. (The HI line frequency is 1420.4058 MHz).

   You can change the l.o. frequency in software: use `set_hp`.

   The line is weak, and you'll need to take lots of spectra. Recall that `getpico` obtains a specified number of spectra (`nSpectra`) each of which has 32000 datapoints and writes them all out into a single file. Suppose you decide to obtain 10000 spectra. You might be tempted to do it in one fell swoop by setting `nSpectra=10000`. That will work, but be aware that it will take about 5 minutes or more. You might want to instead take ten sets with nSpectra=1000 so you can do a sanity check on your data without waiting for what seems like forever.

2. Intensity calibration requires a second pair of measurements, which can be short. Easiest is probably to take one with the horn looking at a known blackbody and one looking at cold sky. What's a convenient blackbody? You! And your friends! So take one short measurement with the horn pointing straight up at cold sky and the other

with as many people as you can find standing in front of it and 'filling the aperture'. (Or you can use rainwater). Call these spectra $s_{coldsky}$ and s_{300K}.

7.   THE ANALYSIS (First Week; by INDIVIDUALS)

7.1.   Take a Suitable Average/Median

Consider, first, the s_{online} and $s_{offline}$ spectra, from which you can find the line shape. There are many individual spectra, 10000 in our above example. You need to combine these to make a single spectrum for each measurement. You can do this by averaging the power spectra (use IDL's `total` function) or by taking the medians (use IDL's `median` function). The former gives a less noisy result, but the latter handles time-variable interference better; use both and compare the results. Even after combining the 10000 spectra, the resultant spectrum will still look noisy. You can reduce the noise by averaging over channels—by 'smoothing' the spectrum. This reduces the noise, but degrades the spectral resolution, so you have to make a compromise on how many channels to smooth over. To decide, realize that the HI line is never narrower than about 1 km/s, so it's OK to degrade the frequency resolution to, say, 1 or 2 kHz. Again, you do the smoothing by averaging (by using IDL's `smooth` function) or medianing (by using IDL's `median`), and again try both to see what happens.

In the smoothed s_{online} spectrum you might not be able to see the HI line, because the instrumental bandpass dominates the spectrum shape. The instrumental bandpass is determined mainly by the low-pass filter, which should fall smoothly to zero as the frequency increases. Does it? If not, should you worry about aliasing?

7.2.   Get the Line Shape

You can remove the instrumental bandpass, and thus get the shape of the line (but not the intensity), by taking the ratio

$$shape = \frac{s_{online}}{s_{offline}} \ . \tag{1}$$

This is the *first* factor (i.e., the shape) in equation (13) in the spectral-line handout.

7.3.   Get the Line Intensity

To get the line intensity in terms of the calibration noise source, multiply the shape spectrum by $T_{sys,coldsky}$—the second factor in the handout's equation (13). We obtain $T_{sys,coldsky}$ by combining the 300K and caloff spectra as linear proportions, as per equation (15) in the handout:

$$T_{sys,coldsky} = \frac{\sum s_{coldsky}}{\sum(s_{300K} - s_{coldsky})}(T_{cal} - T_{sys,coldsky}) \tag{2}$$

Here, a sum is over all channels in a spectrum. All this equation does is to convert measured units, which are digital numbers from the system, into physically meaningful units, i.e. Kelvins. Here, $T_{cal} = 300$ K, because that's the thermal power you injected by standing in front of the horn. Since $T_{coldsky} \ll T_{cal}$, to a first approximation you can neglect it (which is what we did in the handout). Then the final, intensity-calibrated spectrum is equation (16) in the handout, namely

$$final\ calibrated\ spectrum = shape \times T_{sys,coldsky} \tag{3}$$

7.4.   Plotting Intensity vs. Frequency—and Velocity

First, plot your final calibrated spectrum versus the r.f. frequency. Next, plot it versus the Doppler velocity[4]. Remember that, by astronomical convention, positive velocity means motion away (remember the expansion of the Universe!), so

$$\frac{v}{c} = -\frac{\Delta f}{f_0}\ , \tag{4}$$

where c is the speed of light and Δf is the frequency offset from the line frequency f_0[5].

7.5.   Finally, Choose a Reference Frame

You may think we've done it all by this point, but we haven't! We need to correct the observed velocity for the orbital velocity of the Earth, and also the Earth's spin. And

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[4]Astronomers usually express velocities in km s^{-1}.

[5]Radio astronomers, being frequency-oriented, use this equation; optical astronomers, of course, use something different: $\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$. At high redshifts the difference becomes significant; the optical definition is the usual standard.

when observing the Galaxy, it is customary to express velocities with respect to the 'Local Standard of Rest' (LSR), so that's yet another correction.

Calculate the Doppler correction using `ugdoppler.pro` (for documentation from the IDL prompt, use `doc` or `doc_library`). Correct the velocities and plot the spectrum for two reference frames: (1), the barycentric reference frame, which gives velocity with respect to the center of mass of the Solar system[6]; and (2) the Local Standard of Rest (LSR), which is approximately the frame that would rotate around the Galaxy in a circular orbit.

Notes on `ugdoppler`:

1. You need the celestial coordinates of the source, (ra, dec). How to find these for the horn pointing straight up? You could use rotation matrices. However, when looking straight up you don't need this powerful technique because, quite simply, the Dec is equal to the Latitude and RA=LST.

2. You need the Julian day of the observation; see §4.

3. You need the observatory coordinates (north latitude and west longitude) in degrees; you could enter them with the pair of optional input parameters (`nlat, wlong`), but you don't have to because the default values are Campbell Hall's values (which are `nlat= 37.8732, wlong= 122.2573` degrees).

8. IN THE LAB: YOUR SECOND 21-cm LINE MEASUREMENT (Second Week; by GROUPS)

Obtain a fully-calibrated spectrum for the horn pointing at Galactic coordinates $(l, b) = (120°, 0°)$. How do you know where to point the horn? You definitely do need to use rotation matrices! See the handout "Ay120a,b Cheat-Sheet for Spherical Coordinate Transformation" (§5). Plot your spectrum versus both the Barycentric and the LSR velocity.

Your spectrum will probably look something like that in the top panel of Figure 1. There is a zero offset, a curvy 'baseline', and one or more spectral peaks. The curvy baseline comes from nonflat instrumental response before the first mixer, and can be removed with a low-order polynomial fit to the off-line channels (the approximate frequency ranges $-1.2 \rightarrow -0.6$

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[6]There's a third system, the Sun system, which gives velocity with respect to the center of the Sun. The barycenter lies inside the Sun, but not quite at the center, and the barycentric and Sun systems are almost the same.
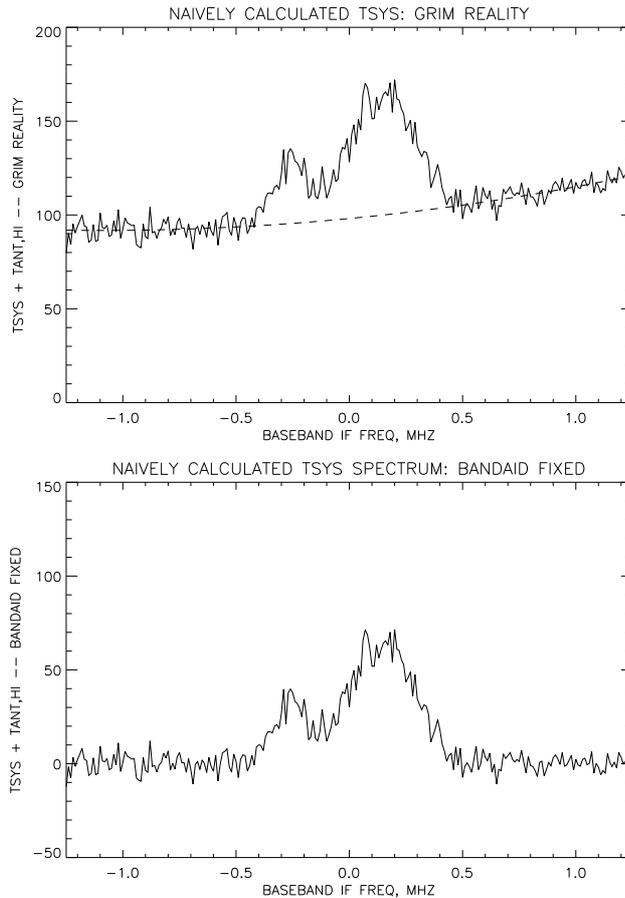
Fig. 1.— A typical raw spectrum with a curvy baseline and multiple velocity components.

and $0.6 \rightarrow 1.2$ MHz). The dashed line is a second-order polynomial fit to the off-line channels and the bottom panel has this fit subtracted off. Here, the multiple peaks come from HI clouds having different velocities. The peaks are usually represented as Gaussians with appropriate central intensities, velocities, and widths. Fitting Gaussians requires specifying initial guesses for the component parameters. These guesses need not be highly accurate. For example, in the bottom panel of Figure 1, initial guesses for the two heights, centers, and widths (FWHM) could be [20, 50], [-0.3, 0.2] MHz, and [0.01, 0.03] MHz. After fitting, check to see if the result looks good: if it does not, then it's usually because you need one or more additional components. It is common to require a low-level, broad component that produces no obvious peak; here, the initial guesses might be [20, 50, 10], [-0.3, 0.2, 0.] MHz, and [0.01, 0.03, 0.07] MHz.

The procedures you need for these least-squares fits are:

1. For the polynomial fit, use either IDL's `poly_fit` or our home-grown `polyfit`.[7]

2. For fitting multiple Gaussians, use our home-grown `gfit`; for evaluating the Gaussians from the parameters produced by `gfit`, use `gcurv`.

In class we'll compare the results from all groups. In this class, we have never yet—after more than a decade's worth of measurements—had the results of all groups agree!

9. IN THE LAB: PROPAGATION VELOCITY MEASUREMENTS WITH SLOTTED LINES AND WAVEGUIDES (Second Week; by GROUPS)

How do you move power or electronic information from one place to another? Transmission lines (cables) and waveguides are indispensable. Sounds easy—just connect two things with a wire. But does this really work? We'll explore cables, waveguides, and reflections by measuring the Voltage Standing Wave Ratio (VSWR), which results from interference of incident and reflected waves.

The experimental work and measurements described below should be done by groups. The analysis in §10 should be done by individuals. The analysis is nontrivial, so don't delay with the measurements!

9.1. The coax slotted line at C-band (~ 3 GHz)

Set up the slotted coaxial line with the wide-range HP 83712B synthesizer. With the far end of the slotted line open, find the wavelength in the slotted line λ_{sl} by measuring the positions of *all* nulls as accurately as you can. Do the same with the far end shorted and note the differences. In particular, note how the positions of the nulls change when the slotted line is terminated with an open versus when it is shorted. Why is this?

From your measurements calculate the velocity [what does "velocity" mean?] in the slotted line by comparing λ_{sl} and the frequency. First do a quick calculation using your measurements. Then do it as accurately as possible by using least-squares fitting and following the advice in §10.1. *Note:* As it happens, the velocity is independent of frequency for a coax cable—in contrast to a waveguide. If you are so motivated, you can check this experimentally.

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[7]Remember that documentation for native IDL procedure is invoked from IDL by typing `?poly_fit`, and for non-native ones `doc, 'polyfit'`.

9.2.   The X-band waveguide (7 to 12 GHz)

Now we'll do the same as we did in §9.1, but for X-band waveguide. The cutoff frequency for X-band waveguide is about 7 GHz. Pick a suitable frequency and, with the far end of the waveguide completely open, measure the VSWR; then short the end of the waveguide, maybe with a piece of aluminum foil or a metal plate, and repeat. Why is an open ended waveguide different from an open ended coax cable?

With the end shorted, measure the velocity in the slotted waveguide by comparing the wavelength and the frequency; do this for several (half-dozen?) frequencies, including one or two near cutoff, and compare with theory. These frequencies should be reasonably closely separated, say by no more than 1 GHz. From these measurements, derive the cutoff wavelength (and thus the waveguide width) as accurately as you can. Also measure the waveguide dimensions, and compare with the width you derived above.

10.   GETTING THE MOST OUT OF YOUR DATA: STATISTICAL ERROR ANALYSIS (Second Week; by INDIVIDUALS)

10.1.   The coax slotted-line wavelength

For the measurements of §9.1, you sampled M cycles of the standing wave in the slotted line. You can calculate the wavelength λ_{sl} from the distances between a single null pair. While this calculation is a good estimate, each of your measurements has an error. You have measured the distance between $(M-1)$ null pairs. What's the best way to combine these measurements so as to obtain the most accurate λ_{sl}?

(1) One way is to calculate the wavelength from each neighboring null *pair* and take the average of the $(M-1)$ *pair*s. You might then apply the usual rule to obtain the uncertainty in the average, i.e. the uncertainty is $\frac{(standard\ deviation)}{(M-1)}$. However, this would *not be correct*. Why? (Hint: The answer you get by averaging the $(M-1)$ pairs is identical to that obtained from a *single* well-chosen non-neighbor pair. Which one?). Actually, this method gives *almost* the best estimate of the wavelength. But you can do better!

(2) The positions of the nulls should increase linearly with distance. So if x_m is the position of null number m, we should have

$$x_m = A + m\frac{\lambda_{sl}}{2} \ , \tag{5}$$

You know x_m and m from measurement and you would like to derive A and λ_{sl}. This is a classic least squares problem; you're fitting a first-degree polynomial, where the unknowns are A and λ_{sl}. You can use our home-grown `polyfit` or the native IDL `poly_fit`.

10.2.   The X-band waveguide

RWvD provide a beautiful discussion of the TE$_{10}$ mode in common rectangular waveguide (in which the ratio of side lengths equals 2). Most important is their equation 11, which gives the "guide wavelength" λ_g; we reproduce it here:

$$\lambda_g = \frac{v_p}{f} = \frac{\lambda_{fs}}{[1 - (\lambda_{fs}/2a)^2]^{1/2}} \;,\qquad (6)$$

where λ_{fs} is the free-space wavelength. The guide wavelength is the wavelength within the waveguide, and combining it with the frequency gives the propagation velocity in the guide.

Here we want to do a least-squares fit of equation 6 to solve for the waveguide width a. Now, you've already measured a with the caliper. But here the idea is to compare your measurements of the waveguide cutoff with what theory (that's equation 6) predicts. So you imagine that a is unknown and determine it from your data on λ_g versus the free-space wavelength λ_{fs}. Doing this is not as straightforward as above in §10.1 because λ_g depends *nonlinearly* on a. The easiest way to handle this problem is by "brute force" method, which we will discuss in class.

From your above solution, determine the best-fit value of a and compare it with the value you measured with the caliper.

11.   IN THE MIND: REFERENCE READING

You can do this lab without doing any reading, so this section, which recommends parts of the highly readable and well-written text by Ramo, Whinnery, and van Duzer (RWvD), *Fields and Waves in Communication Electronics*, is optional. If you have the time and inclination to read, we suggest that you concentrate on the starred items below (***). The notes below refer to the second edition, which should reside in our lab's bookshelf. The italicized numbers refer to the very same sections of the book.

11.1.   Theory of Coaxial Cable

*** *RWvD §5.2: Introduction: what transmission lines do.* Basic equations for transmission lines. Note equation (14) for the characteristic impedance Z_0, which is the ratio of voltage to current in the line. This is expressed in ohms, but it is not "ordinary resistance" because voltage and current are out of phase. Example 5.2 calculates Z_0 for a coax line. RG58/U, done in the example, has 50 ohms; so do the cables we use in our system (in contrast, the TV standard is 75 ohms). Because of the dielectric in the cable, the wave (phase) velocity is ~ 0.7 that in free space, making the wavelength in the cable shorter than that in free space; one needs to account for this when using cables to produce delays in a signal, as in an interferometer or a quadrature circuit.

*** *RWvD §5.4: Reflection and transmission at a resistive discontinuity.* The purpose of a transmission line is to take power from a source to a load. The load should absorb the power and therefore should be resistive. If its resistance matches the line impedance, then all the power transmitted by the line is absorbed by the load. If not, some is reflected; see their equations (4) and (6). We need to tune our devices so that they are "matched" and reflect no power.

11.2.   The Theory of Waveguide

*** *RWvD §8.1: Introduction: general description of waveguides.*

*** *RWvD §8.8: The TE_{10} Mode in a Rectangular Guide.* A nice, complete discussion of the TE_{10} mode in common rectangular waveguide (in which the ratio of side lengths equals 2). In this mode, E is perpendicular to the long side of the guide (and also to the longitudinal axis of the guide). We speak of the polarization of the waveguide: the guide transmits linear polarization that is perpendicular to the long side of a waveguide. When we couple power into a guide from coax, we do so with a probe that is parallel to the internal electric field. When we use a sliding probe to measure VSWR in a waveguide, the probe extends through a long slot in the broad side of the waveguide and the probe is parallel to the E field inside the guide, so it samples the internal E field. Note **Figure 8.8**, which shows the current flow in the waveguide walls; note that the current flow down the center of the broad side of the guide is parallel to the axis of the guide—that is, parallel to the slot that we use for the probe. The slot, being parallel to the current flow, does not interrupt any current flow and has no effect on the internal distribution of fields in the guide. Clever!

RWvD §8.10. Makes the analogy between coax cable and waveguide. A coaxial line can be considered as a waveguide containing walls not only the outside but also on the inside.

The usual mode that propagates in coax is the TEM mode. A TE_{10}-like waveguide mode enters for wavelengths smaller than that given by their equation (4); at this point, coax lines become basically unusable. For type N connectors, we are beginning to approach this limit in our 12 GHz system!

RWvD §*8.11.* Brief description of how coax lines are coupled to waveguide. The method depends on which mode you wish to excite. For the TE_{10} mode, method (b) in Figure 8.11 works well and is almost universally the one used.