MAP PROJECTIONS: REPRESENTING A SPHERICAL SURFACE ON A FLAT PLANE

You make a map on the spherical sky and want to present it on a flat computer screen or sheet of paper. Unless your area is small, or unless you publish on the spherical version of *Ap.J*, you can't avoid dealing with the resulting distortions.

1. SOME IMPORTANT CONCEPTS

1. Preserving shapes; this means preserving angles. Obviously a good idea if you want to represent objects to the eye faithfully. You can only do this "locally". Projections that preserve shapes are called *conformal*. There is no such thing as a perfectly conformal projection, i.e. one that is conformal over large ranges in angle.

2. A *perspective* projection is what you'd see by placing your eye at one position, at the single focal point from which a line emanates through the spheridcal surface and ends up on the projecting surface. For example, if you place your eye at infinity, it's like looking at a globe from far away; this is called the *orthographic* projection.

3. Scale. Scale is the conversion from angle to linear distance on the map ("inches per mile") on an Earth map. The scale of the projection always changes as you move around.

4. Equal-area pixels. The pixels in some projections have equal areas (solid angles), which is necessary for statistical disciussioons. However, such projections are among the worst regarding conformality.

Clear conversion formulae are given by `mathworld.wolfram.com`

2. HEALPix: NOT A PROJECTION

HEALPix (Hierarchical Equal Area isoLatitude Pixelisation of the sphere) is not a projection. Rather, it is a spherical pixelization scheme that is excellent for statistical analysis, and is used by many CMB investigations, including WMAP. It was initially developed by Gorski, Wandelt, and Hivon. The sky is hierarchically tessellated into curvilinear quadrilaterals. The lowest resolution partition is comprised of 12 base pixels. Resolution of the tessellation increases by division of each pixel into four new ones. Areas of all pixels at a given resolution are identical. Pixels are distributed on lines of constant latitude, facilitating applications involving spherical harmonics. A nice discussion and suite of Fortran, IDL, and Python software tools for working with HEALPix format maps is available from
`http://healpix.sourceforge.net/`

3. AZIMUTHAL PROJECTIONS

For an *azimuthal* projection you place a plane tangent to the sphere. The point of intersection is center of the map. For the Earth the easiest example is the center at the North pole, but the center can be anywhere. Most are perspective projections. *None* have equal area pixels. In the maps below, the circles are 15° apart. Important azimuthal projections include:

1. *Orthographic*: The focal point (the eye) is at infinity. This is not conformal: think about looking at a globe, the countries near the edge are very compressed radially.

2. *Gnomonic*: The focal point is the center of the sphere. This not conformal: think about where the equator falls at infinity!). The advantage: straight lines in the projection are great circles on the sphere, so it's really good for airplane pilots.

3. *Stereographic*: The focal point is at the opposite "pole" of the sphere. This *is a conformal projection* and is therefore highly desireable for astronomy. For a hemisphere, the scale increases by a factor of two from center to edge, which is quite good.

4. *Azimuthal Equidistant*: This is *not* a perspective projection. You simply make the radial scale constant everywhere. This is not conformal. Big advantage: you can easily read off the position with a ruler.

PSEUDOCYLINDRICAL PROJECTIONS

Cylindrical projections have the piece of map paper wrapped around a great circle of the sphere. The projection is made and the paper is unwrapped and published in the flat-earth *ApJ*. These are usually chosen to have equal area pixels, which makes them attractive for the rigorously-scientifically-minded crowd who appreciates statistics more than the mental image and aesthetics. None are perspective projections. None are conformal; in fact, distortions are *really bad*.

1. *Aitoff*: The entire sphere fits onto an ellipse with 2:1 axial ratio.

2. *Mollweide*: Much like the Aitoff. The entire sphere fits onto an ellipse with 2:1 axial ratio.

3. *Sinusoidal*: The horizontal length is the cosine of the distance from the equator, which makes the pixels equal area.

4. *Cylindrical equidistant*: A simple cylinder with latitude scale constant. Transformations are easy and can be measured with a ruler. Useful for projections near the equator. If the latitude is stretched to make equal area pixels, then it's called "equivalent cylindrical".

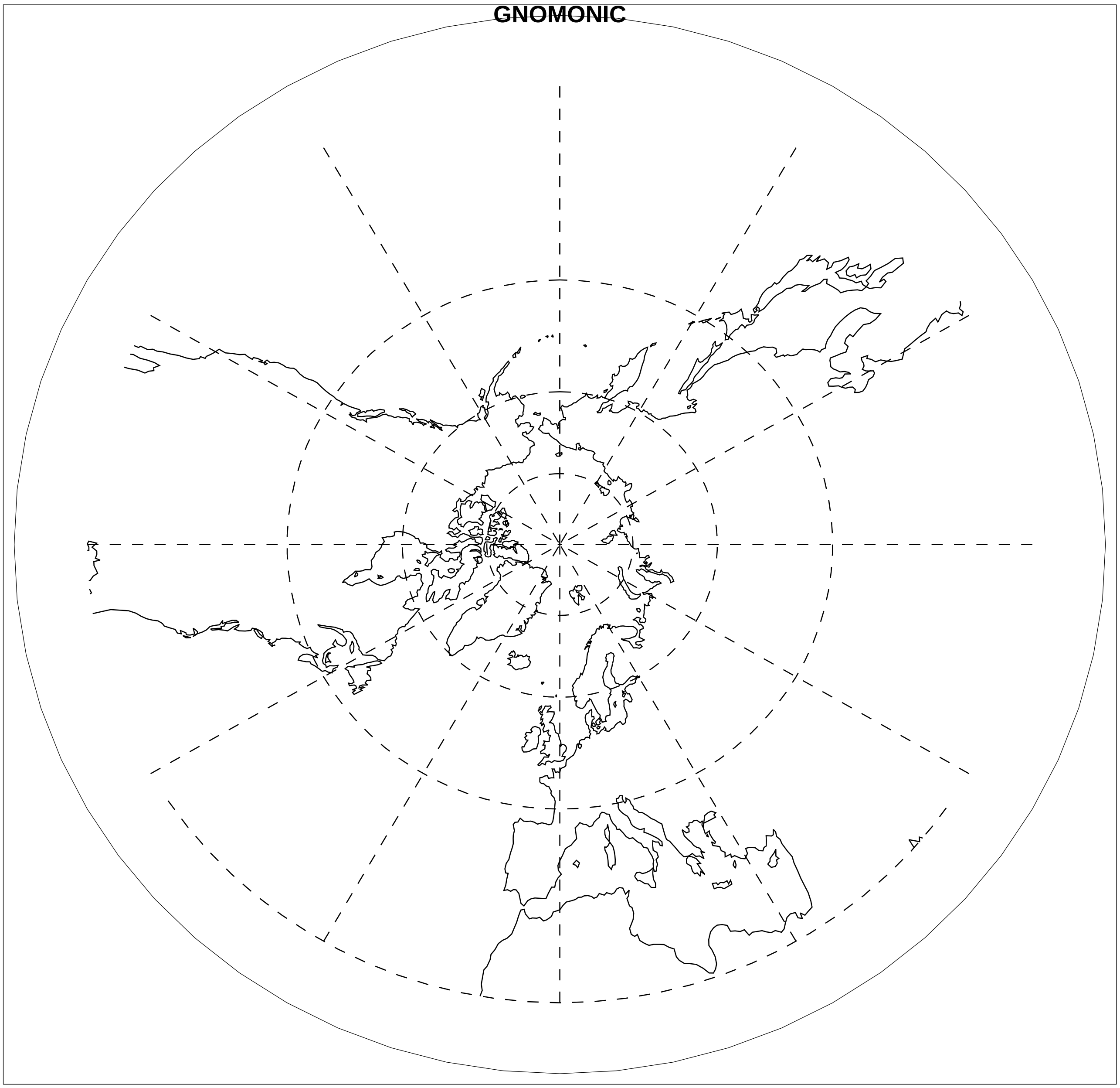

Fig. 1.— The Earth in Gnomic projection. The focal point (the eye) is the center of the sphere. This not conformal: think about where the equator falls at infinity!). The advantage: straight lines in the projection are great circles on the sphere, so it's really good for airplane pilots.

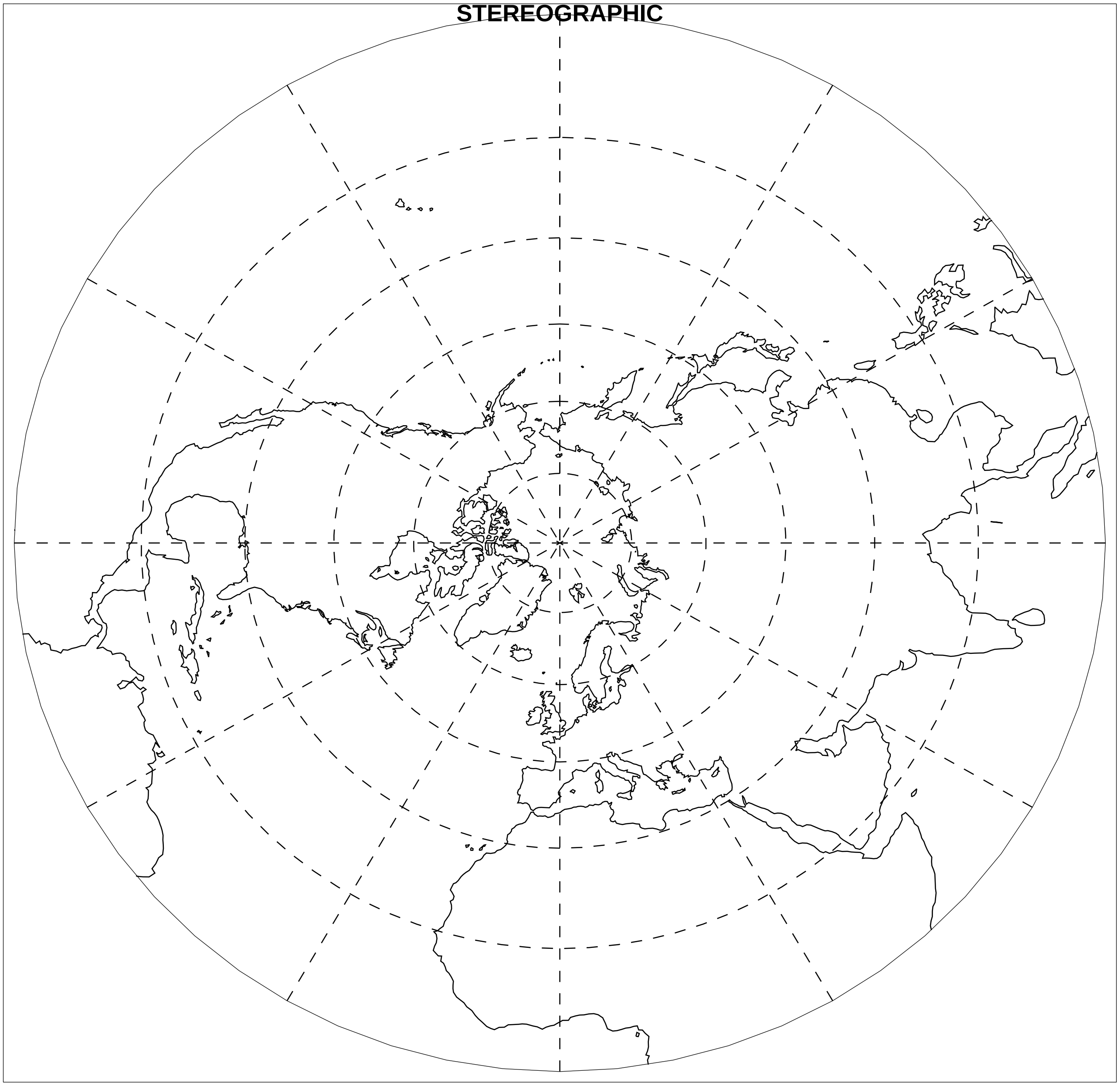

Fig. 2.— The Earth in Stereographic projection. The focal point is at the opposite "pole" of the sphere. This projection is as conformal as they get; in particular, small circles project as small circles (although the centers are displaced). This conformality makes this projection good for visualization and therefore highly desireable for astronomical images. For a hemisphere, the scale increases by a factor of two from center to edge, which is a modest change comopared to many projections.

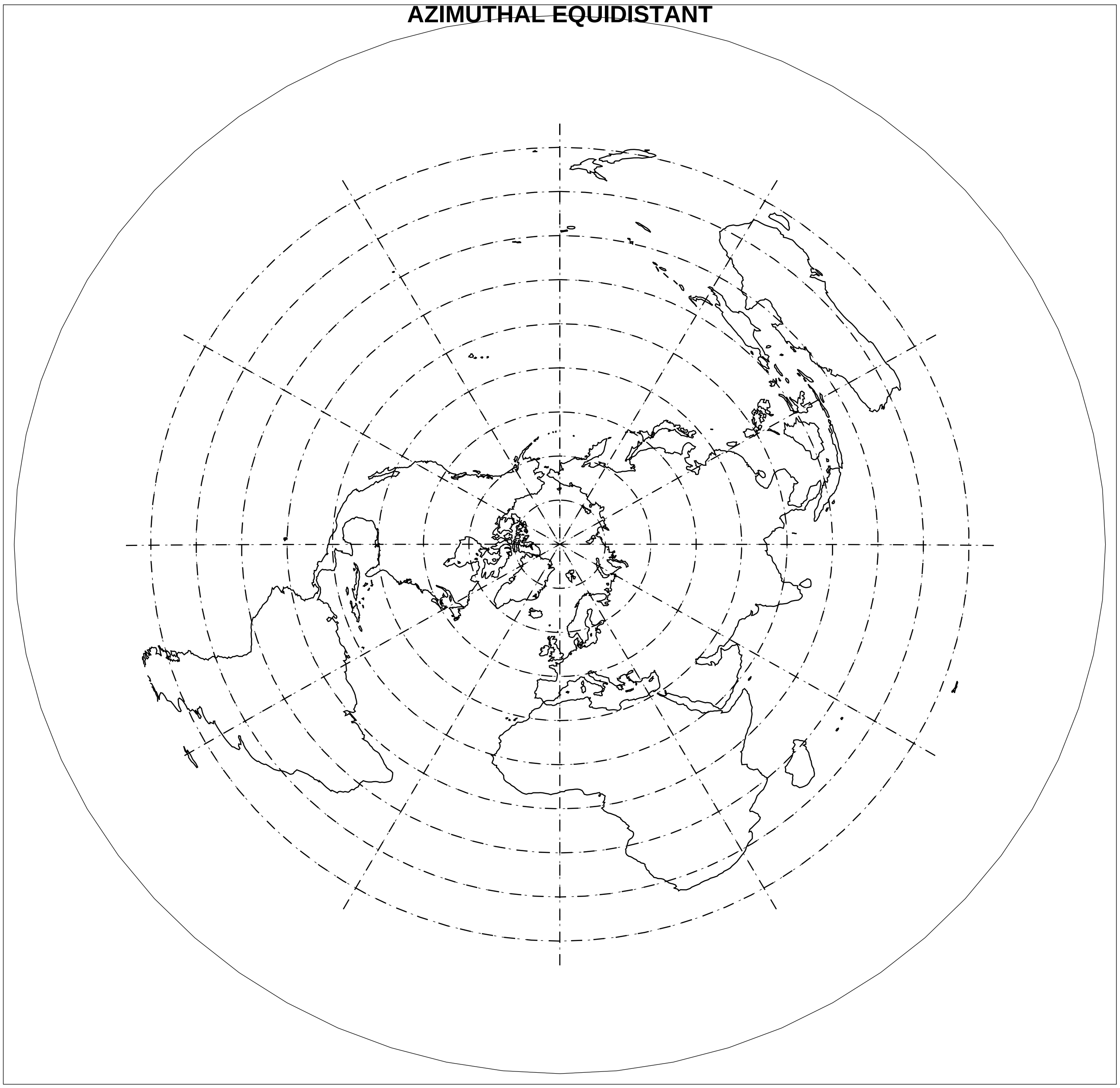

Fig. 3.— The Earth in Azimuthal Equidistant projection. This is *not* a perspective projection. You simply make the radial scale constant everywhere. This is not conformal. Big advantage: you can easily read off the position with a ruler.

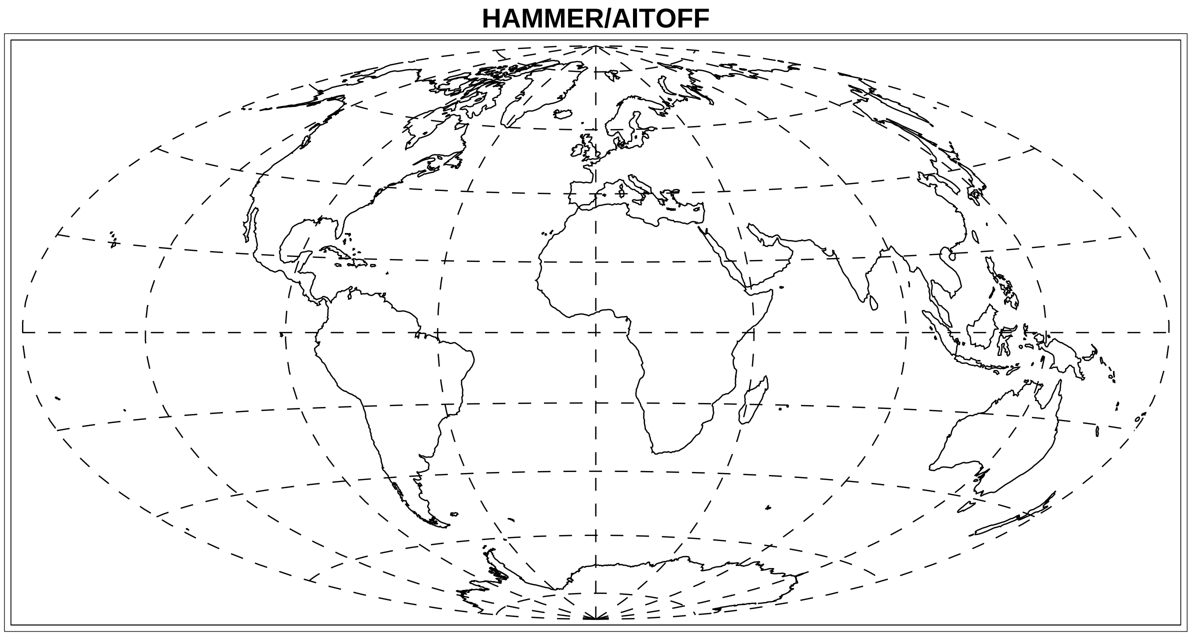

Fig. 4.— The Earth in Aitoff projection.

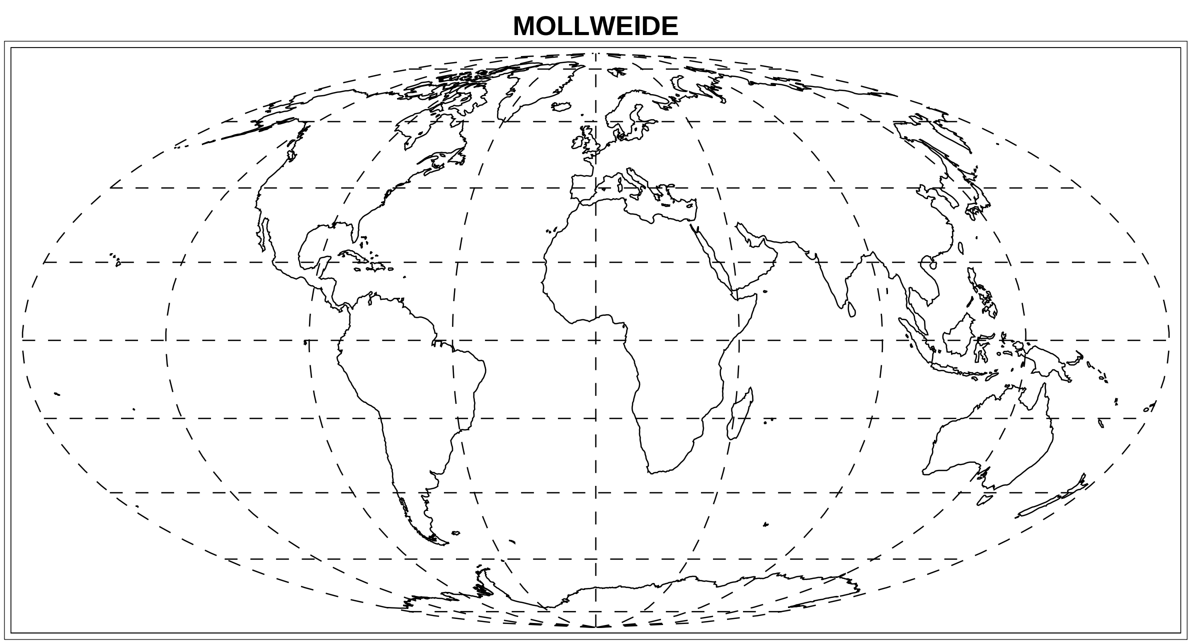

Fig. 5.— The Earth in Mollweide projection.

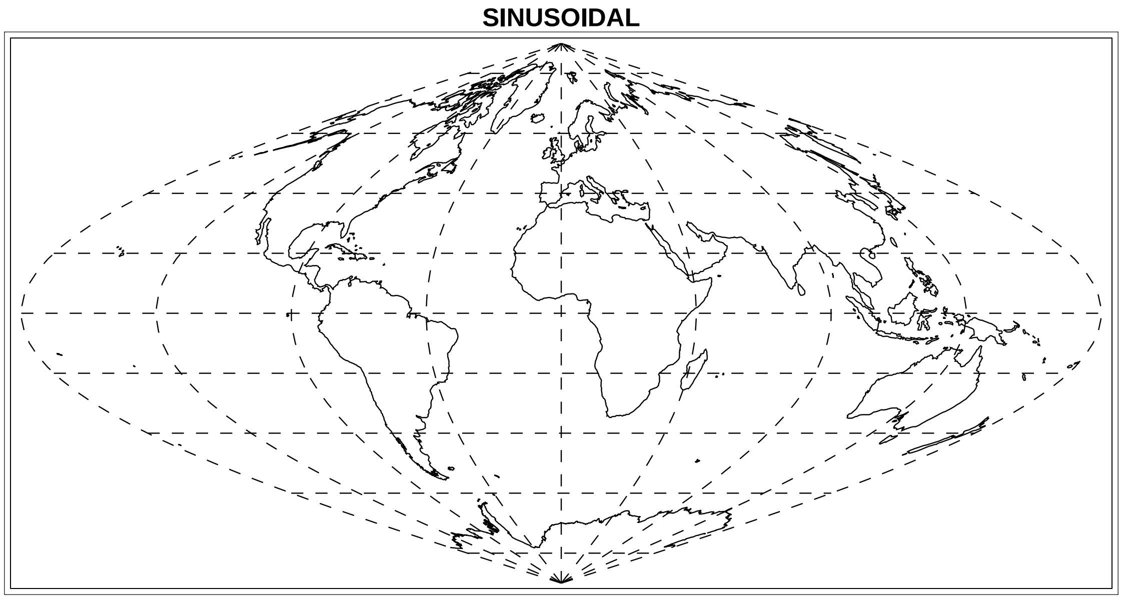

Fig. 6.— The Earth in Sinusoidal projection. The horizontal length is the cosine of the distance from the equator, which makes the pixels equal area.

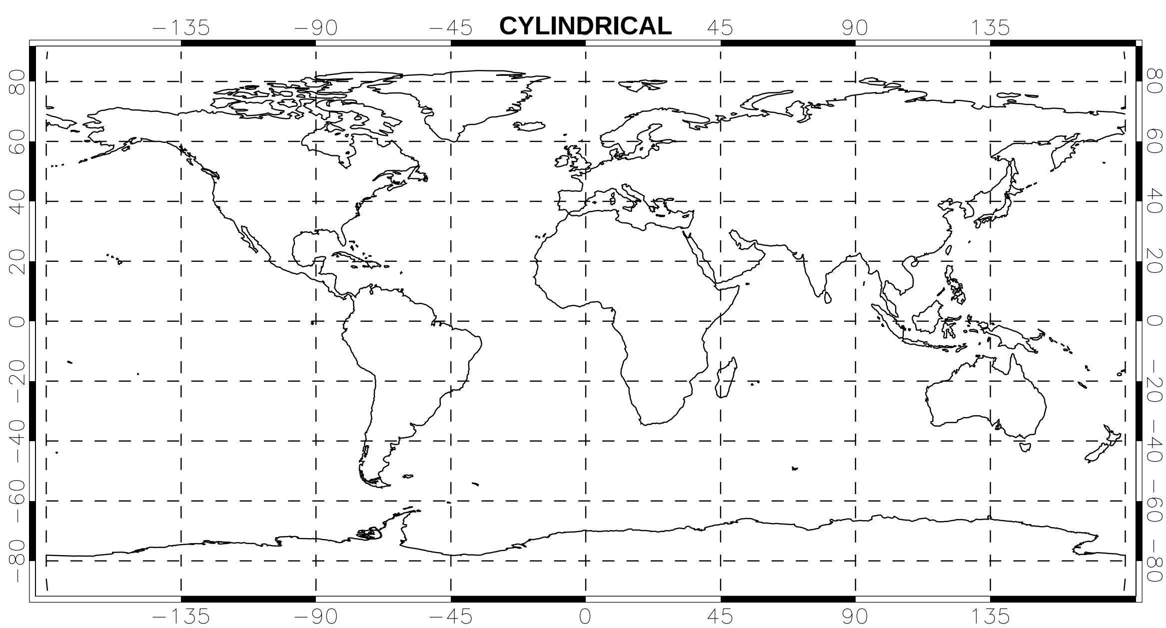

Fig. 7.— The Earth in Cylindrical projection. A simple cylinder with latitude scale constant. Transformations are easy and can be measured with a ruler. Useful for projections near the equator. If the latitude is stretched to make equal area pixels, then it's called “equivalent cylindrical”.