Spectrograph Basics

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Dispersive Spectrometers

- Dispersive spectrometers are a class of instruments that encode wavelength as position on a focal plane detector
- Dispersion can be caused by refraction or diffraction
- Key element is
  - Prism \( dn/d\lambda \neq 0 \)
  - Grating
- Gratings are favored
  - Flexible
    - Transmission or reflection
    - Groove spacing
    - Plane or powered surface
  - Efficient
    - Grating can be blazed
  - Lightweight
Spectrometers as Imagers

• A spectrometer is fundamentally a device which makes an image of a source
  – The position of the image of the source depends on wavelength
• Typically the spectrometer makes an image of an aperture or slit
  – In the solar spectrometer, the spectrometer makes an image of the light exiting an optical fiber
• The location and size of the image is determined jointly by the laws of geometric optics and the grating equation
Fiber source \( d = 100 \, \mu m \)

Filter wheel

Collimator \( f = 179 \, mm \)

Grating \( \theta_g = 64.5^\circ \)
\( \sigma = 12.5 \, \mu m \)

Camera \( f = 200 \, mm \)

CCD 13 \( \mu m \) pixels

2\( \theta = 10^\circ \)

Fiber source \( d = 100 \, \mu m \)

Collimator \( f = 179 \, mm \)

Grating

Camera

CCD

Fiber source
For a reflection grating $\alpha$ and $\beta$ have the same sign if they are on the same side of the grating normal. For a transmission grating they have the same sign if the diffracted ray crosses the normal.

$$m\lambda = \sigma(\sin \alpha - \sin \beta)$$

- for transmission
+ for reflection
Condition for Constructive Interference

\[ m\lambda = \sigma (\sin \alpha \pm \sin \beta) \]
Orders

Transmission grating $1/\sigma = 600 \text{ mm}^{-1}$
Positional Encoding

- The angle is given by the grating equation and the position set by the camera focal length

\[ m \lambda = \sigma (\sin \alpha + \sin \beta) \]
\[ \beta = \arcsin\left(\frac{m \lambda}{\sigma} - \sin \alpha\right) \]
\[ p = f_{cam} \tan(\beta) \]
Mapping Wavelength to Angle

- Holding $\alpha$ and $m$ constant, $\beta$ varies with $\lambda$

$$\beta = \arcsin\left(\frac{m\lambda}{\sigma} - \sin\alpha\right)$$
Mapping Wavelength to Position

- Holding $\alpha$ and $m$ constant, $p$ varies with $\lambda$

$$p = f_{\text{CAM}} \tan(\beta - \beta_0)/\Delta p$$

$f_{\text{CAM}} = 200$ mm

$\Delta p = 13$ $\mu$m
Dispersion

- Dispersion gives the angular spread of diffraction, $\delta \beta$, for a source with wavelength spread, $\delta \lambda$
  - Start with the grating equation and hold the angle of incidence, $\alpha$, and the order, $m$, constant
    \[
    m \lambda = \sigma (\sin \alpha + \sin \beta)
    \]
    \[
    m \delta \lambda = \sigma \cos \beta \delta \beta
    \]
    \[
    \left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha, m} = \frac{m}{\sigma \cos \beta}
    \]
Dispersion

- Over a limited range of wavelength dispersion is \( \approx \) constant
  - Linear relation between wavelength & position

\[
\left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} = \frac{m}{\sigma \cos \beta}
\]
Dispersion

- With higher dispersion it is possible to distinguish closely spaced wavelengths
- High dispersion corresponds to
  - High order (large $m$)
  - Narrow grooves/high groove density
  - Large $\beta$ ($\approx \pi/2$)

$$\left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} = \frac{m}{\sigma \cos \beta}$$
Spectral Resolution

\[ \Delta \alpha = \Delta s / f_{\text{col}}, \quad \Delta \beta = \left( \frac{\partial \beta}{\partial \alpha} \right)_{\lambda} \Delta \alpha, \quad \Delta p = f_{\text{cam}} \Delta \beta \]
Spectral Resolution: the Diffraction Limit

• Even if the input is a point source, the image has a finite size on the CCD array, $\Delta p$, due to diffraction
  
  – The angular size of camera images, $\delta\beta = \lambda/D_{\text{cam}}$, limits the spectral resolution

  \[
  \delta\lambda = \frac{\partial \lambda}{\partial \beta} \delta\beta = \frac{\sigma \cos \beta}{m} \delta\beta
  \]

  \[
  \delta\beta = \frac{\lambda}{D_{\text{cam}}}
  \]

  \[
  R = \frac{\lambda}{\delta\lambda} = \frac{(\sin \alpha + \sin \beta)}{\cos \beta} \frac{D_{\text{cam}}}{\lambda} \approx 2 \frac{D_{\text{cam}}}{\lambda} \tan \theta_B
  \]
Spectral Resolution: the Diffraction Limit

- $D_{\text{cam}} = 75$ mm
- $\tan \theta_B = 2$
- $\lambda = 0.632 \, \mu m$

$R_{DL} \approx 470,000$
Slit Limited Spectral Resolution

\[
\Delta \alpha = \Delta s / f_{\text{col}}, \quad \Delta \beta = \left( \frac{\partial \beta}{\partial \alpha} \right)_\lambda \Delta \alpha, \quad \Delta p = f_{\text{cam}} \Delta \beta
\]
Slit Limited Spectral Resolution

• Generally, the source is not a point
  – If the extent is greater than the diffraction blur then the spectrometer resolution “slit limited”

\[
\delta \alpha = \frac{\delta s}{f_{\text{col}}}
\]

\[
\delta \beta = \left( \frac{\partial \beta}{\partial \alpha} \right)_{\lambda,m} \delta \alpha = \frac{\cos \alpha}{\cos \beta} \delta \alpha = \frac{\cos \alpha}{\cos \beta} \frac{\delta s}{f_{\text{col}}}
\]

\[
\delta \beta = \left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} \delta \lambda = \frac{m}{\sigma \cos \beta} \delta \lambda
\]

\[
\frac{m}{\sigma \cos \beta} \delta \lambda = \frac{\cos \alpha}{\cos \beta} \frac{\delta s}{f_{\text{col}}}
\]

Hence, \( R_{\text{SL}} = \frac{\lambda}{\delta \lambda} = \frac{\sin \alpha + \sin \beta}{\cos \alpha} \frac{f_{\text{col}}}{\delta s} \)

Which is bigger \( R_{\text{DL}} \) or \( R_{\text{SL}} \)?