Lab V: SPECTROSCOPIC MEASUREMENT of SOLAR ROTATION USING the DOPPLER SHIFT

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This lab involves observations of the sun. Avoid looking directly at the sun or reflections of the sun. Never look at the sun through a telescope!

1 Introduction

Geoff Marcy records the spectra of stars and uses repeated measurements to search for periodic Doppler shifts that reveal the presence of orbiting planets. The Doppler shift, \( \delta \lambda \), measures the radial component of the star’s velocity

\[
v = \frac{\delta \lambda}{\lambda} c,
\]

where \( c \) is the speed of light.

Since Marcy measures the Doppler shift of the star and not the planet, the amplitude of the signal is small. For two masses, \( m_1 \) and \( m_2 \), in a circular orbit of semimajor axis, \( a \), the orbital velocity is

\[
v_1 = \sqrt{\frac{Gm_2^2}{a(m_1 + m_2)}},
\]

or inserting numerical values for a 1 solar mass star,

\[
v_{\text{star}} = 12.7 \left( \frac{a}{5 \text{ AU}} \right)^{-1/2} \left( \frac{M_{\text{planet}}}{M_{\text{Jupiter}}} \right) \text{ m s}^{-1}
\]

Since the speed of light is \( 3 \times 10^8 \text{ m s}^{-1} \), you might expect that a spectral resolution, \( R \), of

\[
R = \frac{\lambda}{\Delta \lambda} = \frac{c}{v} \simeq 2 \times 10^7
\]
would be required to detect planets. Marcy’s group has recently detected several Neptune-mass ($17 M_\oplus = 0.05 M_{Jupiter}$) planets (Fig. 1). The velocity precision needed to find these objects is about 5 m s$^{-1}$.

![Figure 1: The recent detection of a Neptune-mass (20 $M_\oplus$) planet by Geoff Marcy’s group. Measured velocities vs. orbital phase are plotted for GJ 436 (filled dots), with repeated points shown as open circles. The dotted line is the radial velocity curve from the best-fit orbital solution, $P = 2.644$ d, $e = 0.12$, $M \sin i = 0.067 M_{JUP}$. The RMS of the residuals to this fit is 5.26 m s$^{-1}$ with a reduced $\chi^2 = 1.00$.](image)

The spectrographs used for the Doppler search have $R \approx 50,000$ or a velocity precision of 6 km s$^{-1}$. How is it possible to detect planets with such crude measurements? The answer is that Geoff observes his target stars at very high signal-to-noise and records many hundreds of stellar absorption features. The combination yields a precision must better than 6 km s$^{-1}$.

In this lab we will use similar techniques, not to measure the Doppler shift of other stars, but that of our own sun.

## 2 Measuring the Radius of the Sun

An elementary problem in astronomy is to measure the diameter of the sun. This can be accomplished by comparing the solar rotational period with the
rotation speed of the solar surface. The rotational speed, \( v_{\text{rot}} \) is

\[
v_{\text{rot}} = \frac{2\pi R_\odot}{T_{\text{rot}}}. \tag{5}
\]

The rotation period can be established by noting the motion of sun spots, and at the solar equator the period, \( T_{\text{rot}} \), is about 25 days. The rotation speed can be inferred from the Doppler shift of material in the solar photosphere at the solar limb (the edge of the sun).

Fortunately, at optical wavelengths the solar photosphere exhibits strong absorption features that can be used to measure the Doppler shift. Fig. 2 shows an example of a small section of a spectrum of the sun at high spectral resolution.

![Figure 2: A small section (10 nm) of the solar spectrum at high spectral resolution (\( \lambda/\delta\lambda \approx 10^5 \)) from the McMath-Pierce 1-meter Fourier Transform Spectrometer. Numerous narrow absorption lines are present which make it possible to measure the Doppler shift of stars to high precision.](image)

Data from the solar spectral atlas used to make Fig. 2 are available from
It is the presence of these narrow absorption features that make possible the indirect detection of exoplanets; we will use them to establish the rotation speed of the sun.

From other methods, e.g., transits of Venus, we can guess that the radius of the sun is approximately 700,000 km, in which case the rotation speed at the equator, using Eq. (5) must be about 2 km/s. The non-relativistic Doppler shift $\delta \lambda$ is

$$\delta \lambda = \frac{v}{c} \lambda_0,$$

(6)

where $\lambda_0$ is the rest-wavelength. Given that the speed of light is 300,000 km/s, this order of magnitude estimate suggests that we need a spectral resolution of

$$R = \frac{\lambda}{\delta \lambda} = \frac{c}{v_{rot}} \simeq 150,000$$

(7)

to detect the Doppler shift due to solar rotation. You might suspect that even higher spectral resolution may be necessary, since the Doppler shift is only sensitive to the line of sight component of velocity. The full amplitude of the Doppler effect is seen only exactly at the limb of the sun and when the earth lies in the equatorial plane of the sun. Generally, the Doppler shift will be $v_{rot} \sin \theta \cos \phi$, where $\theta$ and $\phi$ are the angles shown in Fig. 3.

### 2.1 Spectral Resolution

There is a general result in spectroscopy which states that the limiting spectral resolution of an instrument is

$$R = \frac{\lambda}{\delta \lambda} = mN,$$

(8)

where $m$ is the order of interference, and $N$ is the number of interfering beams. For example, suppose we are using a diffraction grating in first order, i.e., $m = 1$ with 100 grooves/mm. By “first order” we mean that the grating is oriented so that there is a one wavelength delay between adjacent diffracted beams. In this case the grating would have to be $150,000/100 \text{ mm} = 1.5 \text{ m}$ long to achieve a spectral resolution equivalent to a 2 km/s
Figure 3: The amplitude of the solar rotation at the equator is $v_{rot}$. The full Doppler amplitude is observed only at the limb, and elsewhere is reduced by a factor of $\sin \theta$ when the earth lies in the plane of the solar equator. The earth lies in the solar equatorial plane on June 5 and December 7. Otherwise there is an angle, $\varphi$ between the direction of rotation and the line of sight, and the observed velocity is $v_{rot} \sin \theta \cos \varphi$.

Doppler shift! While meter-long gratings may be practical for large, expensive research instruments, e.g. Fig. 4, they are certainly not an option for an undergraduate astronomy lab.

The approach we take is analogous to that used in the Doppler searches for exoplanets. The reflex Doppler motion due to an unseen planet is a few meters a second, but high resolution spectrometers used to detect Doppler planets achieve only $R \simeq 50,000$. The meter per second precision is accomplished by determining line positions at modest spectral resolution but at very high signal to noise. Many lines are used simultaneously thus the
ultimate Doppler precision is

$$\delta v \approx \frac{1}{SNR} \frac{1}{\sqrt{M R}},$$

(9)

where $SNR$ is the signal-to-noise in the measurement of individual line positions and $M$ is the number of lines measured. If $R \simeq 50,000$, $SNR = 100$ and $M = 100$, the raw spectrometer resolution of $\delta v = 300,000/50,000 \text{ km/s} = 6 \text{ km/s}$ becomes 6 m/s!

To see the validity of this argument consider the measurement of an emission feature using two pixels, with positions labelled $x_- = -1$ and $x_+ = +1$. If the number of photons detected in $x_-$ is $N_-$ and the number of photons detected in pixel $x_+$ is $N_+$ then the center of light is

$$\bar{x} = \frac{N_+ - N_-}{N_+ + N_-},$$

(10)

and $-1 \leq \bar{x} \leq 1$. Evidently, by error propagation, the error in the $x$ position
$$\sigma_x^2 = \frac{N_-}{(N_+ + N_-)^2} + \frac{N_+}{(N_+ + N_-)^2}$$  \hspace{1cm} (11)$$

or

$$\sigma_x = \frac{1}{\sqrt{N}},$$  \hspace{1cm} (12)$$

where \(N = N_- + N_+\) is the total number of photons counted. Further precision implied by the factor of \(\sqrt{M}\) is justified by noting that if we have \(M\) independent measurements from \(M\) different lines, the standard error reduced by the square root of the number of measurements.

Stated succinctly, the core problem to be solved in this lab is how to measure the centroid of an absorption line to a fraction of a pixel.

### 3 Diffraction Gratings & Spectrometers

The solar spectrometer uses an echelle diffraction grating, which is exceptionally delicate. Under no circumstances touch the grating. The grating cannot be cleaned, so it is imperative to keep dust and other contaminants off the grating. Always wear gloves when making any adjustments on the instrument. Use the sticky mat when entering Rm 705 annex and keep the door closed.

#### 3.1 Basic Relations for a Diffraction Grating

The heart of our solar spectrometer is a diffraction grating. The starting point for this section is the grating equation

$$m \frac{\lambda}{\sigma} = \sin \alpha \pm \sin \beta$$  \hspace{1cm} (13)$$

where \(\alpha\) and \(\beta\) are the angles of incidence and diffraction, \(m\) is the order of interference, \(\lambda\) is the wavelength, and \(\sigma\) is the spacing of the diffracting element. This states the condition for constructive interference. The + and − signs correspond to reflection and transmission gratings respectively.

The free spectral range, \(\Delta \lambda\), is just the difference between the wavelength for two lines in adjacent orders that show up at the same value of \(\beta\), thus

$$m(\lambda + \Delta \lambda) = (m + 1)\lambda$$  \hspace{1cm} (14)$$
Figure 5: The angles of incidence, $\alpha$, and diffraction, $\beta$, are shown for a simple grating with groove spacing $\sigma$.

$$\Delta \lambda = \frac{\lambda}{m}. \quad (15)$$

This circumstance is depicted in Fig. 6. As a consequence a narrow band filter, or “order sorting filter” must be inserted into the beam before the spectrum is recorded.

For an arbitrary groove shape of the diffracting structure the energy is diffracted in an essentially uniform manner into the different orders $m = 1, 2, 3...$, and using any one order gives a low efficiency. Choosing a suitable shape for the periodic structure makes the directions of the constructive interference and specular reflection from the grating coincide for a given wavelength and order; a technique known as blazing. When the incident and diffracted beams satisfy the rules of geometric optics

$$\alpha = \delta + \theta, \beta = \delta - \theta \quad (16)$$

Where $\delta$ is the blaze angle, $2\theta$ is the full angle between the incident and diffracted beams. The order of diffraction on blaze is $m$,

$$m = 2\frac{\sigma}{\lambda} \sin \delta \cos \theta. \quad (17)$$
Figure 6: Spectral lines overlapping in adjacent orders of interference. A grating spectrograph is usable over only a small wavelength range, $\approx \Delta \lambda$ known as the free spectral range. A portion of the image for a source with two emission lines is depicted orders 6 and 7 overlap so that the long wavelength line of $m = 6$ starts to blend into the short wavelength line of $m = 7$. The dotted lines show the envelope of the grating response function for a unblazed grating.

3.2 Spectral Resolution

The angular dispersion is

$$\frac{\partial \beta}{\partial \lambda} = \frac{m}{\sigma \cos \gamma \cos \beta}$$

(18)

From which the spectral resolution is

$$\delta \lambda = \frac{\partial \lambda}{\partial \beta} \delta \beta$$

(19)

or

$$\delta \lambda = \frac{\partial \lambda}{\partial \beta} \frac{\partial \beta}{\partial \alpha} \delta \alpha$$

(20)

The term

$$\frac{\partial \beta}{\partial \alpha} = \frac{\cos \alpha}{\cos \beta}$$

(21)

is known as the anamorphic magnification. The resolution of the spectrograph, $R = \lambda/\delta \lambda$, is given by

$$R = \frac{m \lambda}{\sigma \cos \gamma \cos \alpha \delta \alpha}$$

(22)

Thus the resolution is limited by the angular diameter of the source illuminating the spectrometer. In our case the the source is a nominal $d_{fib} = 100$...
Figure 7: A grating with angled grooves can be used to direct the diffracted light preferentially into one order. The blaze angle, \( \delta \), is measured from the facet normal.

\( \mu \)m diameter optical fiber, which feeds a collimating optic of focal length, \( f_{col} = 180 \) mm, thus

\[
d\alpha = \frac{d_{fiber}}{f_{col}}
\]  

(23)

4 What I am Supposed to be Doing?

Here’s a list of the major steps in this lab:

1. Compute the spectral resolution of the spectrometer;

2. Obtain a spectrum of the Ne lamp and measure the wavelength scale and spectral resolution;

3. Obtain a drift scan of the solar disk and measure the Doppler shift as a function of time;

4. Deduce the rotation velocity of the sun and find the radius.
Figure 8: Schematic layout of a spectrometer using a transmission grating. A fiber source illuminates a collimator lens which produces a parallel beam on a dispersing elements (a transmission grating in this case). The diffracted light from the grating is collected by a camera and focused onto a CCD. The spectrometer makes an image of the fiber source on the CCD. The finite size of the fiber determines the spectral resolution of the system.

4.1 Understand the Spectrometer

Your first task is to understand the parameters of the spectrograph. Based on the properties listed in Table 1 compute the following for a wavelength of 632.8 nm:

- Order of interference on blaze ($m$)
- Free spectral range for this order [nm]
- Minimum and maximum wavelengths of order [nm]
- Angular spread of order [deg]
- Magnification (including the anamorphic term)
- Spectral resolution element\(^1\) [pixels]
- Spectral dispersion [nm/pixel]
- Spectral resolution [$R = \lambda/\delta\lambda$]
- Spectral resolution [km/s]

\(^1\)i.e., the size of the image of the fiber on the CCD
• Width of the order on the CCD [mm]
• Width of the order on the CCD [pixels]

Table 1: Nominal Spectrometer Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groove Density (1/σ)</td>
<td>80 mm⁻¹</td>
</tr>
<tr>
<td>Blaze angle (θ_b)</td>
<td>64.5°</td>
</tr>
<tr>
<td>Fiber diameter (d_fiber)</td>
<td>100 µm</td>
</tr>
<tr>
<td>Collimator focal length (f_col)</td>
<td>180 mm</td>
</tr>
<tr>
<td>Camera focal length (f_cam)</td>
<td>200 mm</td>
</tr>
<tr>
<td>CCD pixel size</td>
<td>13 µm</td>
</tr>
<tr>
<td>2θa</td>
<td>10°</td>
</tr>
</tbody>
</table>

*Full angle between incident and diffracted beam

4.2 Calibration

The spectrometer has several degrees of freedom. There are focus adjustments, which should be changed only when you have been checked out! If you change the focus of the collimation optics, you have to go through a complex alignment procedure to get the beam back into collimation. There are also x/y alignments on these lenses, which in general should not be manipulated. If the image quality is poor the easiest adjustment to make is the focus on the commercial camera lens on the APOGEE CCD. Ask an instructor before you change this setting.

The two adjustments which you may want to make are the rotation angle of the echelle grating table and the order sorting filter. Since the free spectral range of the echelle grating is small (10 nm) a narrow band filter is required to choose which order is passed onto the grating. Currently you can select one of three filters: 633, 590 and 488 nm (all 10 nm wide). For this lab all measurements can be made with the 633 nm filter.

The first step in acquiring data that can be used for astronomical purposes is to calibrate the wavelength scale, measure the spectral dispersion (nm per pixel) and estimate the spectral resolution. This is done by selecting the 633 nm filter and obtaining a spectrum of the Ne lamp. The Ne lamp unit sits
Figure 9: A spectrum of the Ne calibration lamp using the 633 nm blocking filter. The white spots correspond to individual emission lines. The dispersion direction, i.e., wavelength, is along rows. Note that the $x$-axis displays the entire width of the spectrum, but the $y$-axis spans only 100 pixels. You can speed up the data acquisition process by windowing the APOGEE CCD before you read out data.

on the side of the optical bench, and feeds the spectrograph input via a short orange optical fiber. **Note: Optical fibers are delicate! Do not bend them sharply, step on them or pinch them.**

You will only need a short exposure, $\simeq 100$ ms. If you don’t see any emission lines it may be that the grating orientation is wrong. You can check this by illuminating the fiber with white light, e.g., a dimmed reading lamp controlled by the variable transformer. You should see a streak of light running along rows with the brightest section in the middle of the CCD array. The width of the streak is defined by the width of the blocking filter. Again, you will only need a short exposure. If you don’t see any light ask for help! If all else fails we can show you how to use the 632.8 nm laser to align the grating.

Fig. 9 shows an example of the Ne spectrum near 633 nm. You should see a series of bright spots running along CCD rows. The spectrum spans the full width of the APOGEE detector, but is only a few tens of pixels high. You can speed up data acquisition by setting a window in the $y$-direction of about 100 pixels.

Your next step is to identify these spots. You can do this by using the on-line Ne atlas at Kitt Peak National Observatory to identify the lines:

http://www.noao.edu/kpno/specatlas/hnear/hnear.html

An ASCII table of these data is at:

http://www.noao.edu/kpno/specatlas/hnear/hnearhres.dat

Note that both these resources include lines from species other than Ne.
Figure 10: A small section of the Ne lamp spectrum showing a single emission line. A Gaussian profile has been fit to the line showing that the FWHM is about 0.06 nm.

Use these data to establish the wavelength scale of the spectrometer, show that the dispersion ($d\lambda/d\text{pixel}$) is approximately linear. Do this by plotting wavelength as a function of pixel value. Estimate the spectral resolution of the spectrometer in nm and km/s from the measured dispersion and the size of the images of the fiber. Compare your result with Fig. 10. Note that the dispersion direction (increasing wavelength) is not oriented exactly along rows. If the FWHM of your Ne lines is larger than 4-5 pixels, something is amiss.

4.3 Solar Observing

Never look at the sun through any optical instrument! Do not look through the eyepiece or the finding telescope. You must not set up the solar telescope without the aperture mask, and the first time
you use the solar telescope you must do it with Graham, Sandstrom or Sincher.

Carry the Criterion 8-inch up to the N deck of the Campbell Hall roof. You will need to align the mount so that the polar axis of the telescope is oriented parallel to the spin axis of the earth. You can align the telescope N/S by using the degree circle on the roof and the equatorial wedge on the telescope can be adjusted, knowing that the latitude of Berkeley is 37°.

Once the telescope is aligned, retrieve the blue 100 μm fiber from the storage location and attach it to the projection screen. Point the telescope to the declination of the sun and slew the telescope in hour angle until you have an image of the solar disk. Check the telescope focus. Refine the pointing in declination so that an hour angle scan moves the telescope across the broadest dimension of the sun. Make sure that the telescope is balanced, so that the telescope pointing does not sag over time. Take a test exposure with the solar disk on the fiber and choose an exposure time that gives plenty of counts\(20,000 < ADU < 50,000\), but does not saturate.

Point the telescope a few minutes in hour angle to the west of the sun and begin taking CCD frames in continuous readout mode. The rotation of the earth will scan the solar disk over the fiber. Be sure that you have windowed down the readout so that only a few hundred pixels in the \(y\)-direction are read out. Continue taking exposures until the western limb has passed over the fiber for about one more minute.

### 4.4 Measure the Doppler Shift

You now have about 50 to 100 spectra. The early spectra will contain only scattered light. The spectra where the sun is directly over the fiber should be apparent. You need to figure out how to correct your spectra for scattered light and to extract flux as a function of position. You can’t use a single row of data because this will throw away valuable photons, and because the dispersion direction is not oriented exactly along rows.

Once you have extracted these “1-d” spectra you can use the method of cross-correlation to search for the Doppler shift from one spectrum to the next. This cross-correlation is more tricky than you have experience before, because you need to find a fractional pixel offset between spectra!

When you have mastered measuring fractional pixels shifts, you should see a systematic drift in line positions between the E and W limbs. Plot the Doppler shift as a function of time (taken from the FITS header) and
demonstrate this shift.

Compute the coordinates of the observed position on the sun as a function of time and evaluate the geometric projection factor depicted in Fig. 3. Show how your predicted velocity variation as a function of time compares with your observations. Compute the radius of the sun and quote your error bars.